2nd International Congress of Serbian Society of Mechanics (IConSSM 2009)
Palić (Subotica), Serbia, 1-5 June 2009 A-10:1-9

A NOTE ON STABILITY OF HOVERING MAGNETIC TOP

S. Djaković[1], S. Simić[2]

Department of Mechanics, Faculty of Technical Sciences
University of Novi Sad, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia
e-mail[1]: NSanja83@yahoo.com
e-mail[2]: ssimic@uns.ac.rs

Abstract. This note discusses some issues related to stability of stationary motion of magnetized top in homogeneous magnetic field. Stability of synchronous motion is analyzed using the simplified model in which the hovering motion of the center of mass is ignored. Stability boundaries are derived using Lyapunov's direct method. In particular, it is shown that, for a given angle Δ between magnetic moment dipole and principal axis of the top, there is an interval of stationary values of nutation angle θ_0 for which the stationary synchronous motion is stable.

1. Introduction

The Levitron™ is commercial product consisted of a permanent magnetic base and a top which is also a magnetic dipole. It is possible to produce stable levitating motion of the top above the magnetic base, akin to stable precession of a standard gyroscope. Although its inventor, Ray Harrigan, was discouraged by academic community, he persisted in the efforts to construct a hovering top. This remarkable toy attracted the attention of physicists because Earnshaw's theorem rules out stable magnetic levitation of static magnetic dipoles. It called for reasonable explanation of the phenomenon and stability analysis of stationary motion.

The papers of Simon et al. [1] and Berry [2] provided the explanation of the phenomenon of hovering motion of magnetic top. On the other hand, recent studies of Flanders et al. [3], Dullin and Easton [4] and Krechetnikov and Marsden [5] tackle the problem of stability of the stationary hovering motion. These stability results are based upon the analysis of linearized variational equations. Moreover, stability boundaries were determined from the conditions of marginal stability since asymptotic stability neither can be mathematically proven, nor physically achieved. Therefore, a question about the influence of non-linear terms could be posed.

In order to fill this gap, this study presents the results of stability analysis based upon application of Lyapunov's direct method. Stability of synchronous motion is analyzed using the simplified model [3] in which the hovering motion of the mass center is ignored, i.e. the center of mass is treated as a fixed point. It is shown that appropriate Lyapunov function cannot be constructed using energy-like first integral solely, but rather as a linear combination of two fist integrals – generalized energy integral and angular momentum integral. A new result, obtained using this approach, shows that for a given angle Δ between magnetic moment dipole and the principal axis of the top there is an interval of stationary values of nutation angle θ_0 for which stationary synchronous motion is stable.

2. Stationary motion of magnetic top

Governing equations of motion of the magnetic top, with moment **m** of magnetic dipole, in homogeneous magnetic field **H** will be given in the form of Lagrangian equations. It will be assumed that the center of mass of the top is fixed, i.e. its hovering motion will be ignored, and the influence of gravitational force equilibrated by the constant magnetic field. It will also be assumed that moment of magnetic dipole is constant, fixed in the body and forms constant angle Δ with the axis of symmetry of the top. Introducing standard Euler's angles of precession, nutation and rotation (ψ, θ and φ), vector **m** could be expressed as

$$\mathbf{m} = m\cos\Delta\,\boldsymbol{\nu} - m\sin\Delta\,\boldsymbol{\mu}; \quad m = \text{const.},$$

and magnetic field is assumed to be constant, having vertical direction

$$\mathbf{H} = -H\mathbf{k} = -H\left(\sin\theta\,\sin\varphi\,\boldsymbol{\lambda} + \sin\theta\,\cos\varphi\,\boldsymbol{\mu} + \cos\theta\,\boldsymbol{\nu}\right); \quad H = \text{const.},$$

where $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ are unit vectors of the moving frame $O\xi\eta\zeta$ (see Fig. 1). Corresponding potential energy of magnetic field reads

$$\Pi_{mag} = -\mathbf{m}\cdot\mathbf{H} = mH\left(\cos\Delta\,\cos\theta - \sin\Delta\,\sin\theta\,\cos\varphi\right).$$

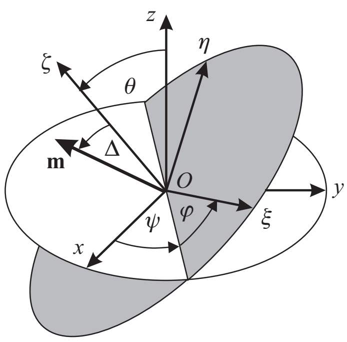

Figure 1. Coordinate frames of the hovering top problem.

It is assumed that the body is axially symmetric, with axial moments of inertia $J_\xi = J_\eta \neq J_\zeta$, so that Lagrangian function of the magnetic top in homogeneous magnetic field can be written in the form

$$L = E_k - \Pi_{mag} = \frac{1}{2}J_\xi\left(\dot\theta^2 + \dot\psi^2\sin^2\theta\right) + \frac{1}{2}J_\zeta\left(\dot\varphi + \dot\psi\cos\theta\right)^2 - mH\left(\cos\Delta\,\cos\theta - \sin\Delta\,\sin\theta\,\cos\varphi\right), \tag{1}$$

where an overdot denotes time derivative. Lagrangian equations $(d/dt)(\partial L/\partial\dot q) - \partial L/\partial q = 0$ for coordinates $q = \psi, \theta, \varphi$ read

$$\ddot\psi\left(J_\xi\sin^2\theta + J_\zeta\cos^2\theta\right) + J_\zeta\ddot\varphi\cos\theta + (J_\xi - J_\zeta)\dot\psi\dot\theta\sin 2\theta - J_\zeta\dot\varphi\dot\theta\sin\theta = 0; \tag{2}$$

$$J_\xi\ddot\theta - \dot\psi^2(J_\xi - J_\zeta)\sin\theta\,\cos\theta + J_\zeta\dot\varphi\dot\psi\sin\theta - mH\left(\cos\Delta\,\sin\theta + \sin\Delta\,\cos\theta\,\cos\varphi\right) = 0; \tag{3}$$

$$J_\zeta\left(\ddot\varphi + \ddot\psi\cos\theta - \dot\psi\dot\theta\sin\theta\right) + mH\sin\Delta\,\sin\theta\,\sin\varphi = 0. \tag{4}$$

Note that governing equations have two first integrals, Jacobi (generalized energy) and cyclic integral since $\partial L/\partial t = 0$ and $\partial L/\partial \psi = 0$. They have the following form:

$$I = \frac{1}{2}J_\xi\left(\dot\theta^2 + \dot\psi^2\sin^2\theta\right) + \frac{1}{2}J_\zeta\left(\dot\varphi + \dot\psi\cos\theta\right)^2 \tag{5}$$
$$+ mH\left(\cos\Delta\cos\theta - \sin\Delta\sin\theta\cos\varphi\right) = \text{const.}$$
$$C = \left(J_\xi\sin^2\theta + J_\zeta\cos^2\theta\right)\dot\psi + J_\zeta\dot\varphi\cos\theta = \text{const.} \tag{6}$$

The results of this study will be presented in dimensionless form. To that end the following dimensionless quantities will be introduced

$$J = J_\xi/J_\zeta; \quad \tau = t\hat\omega; \quad \hat\omega = \sqrt{mH/J_\zeta},$$

where $\hat\omega$ is reference quantity which has dimension s^{-1}. Using these quantities Lagrangian equations (2)-(4) can be given in dimensionless form

$$\psi''\left(J\sin^2\theta + \cos^2\theta\right) + \varphi''\cos\theta \tag{7}$$
$$+ (J-1)\psi'\theta'\sin 2\theta - \varphi'\theta'\sin\theta = 0;$$
$$J\theta'' - \psi'^2(J-1)\sin\theta\cos\theta + \varphi'\psi'\sin\theta \tag{8}$$
$$- \left(\cos\Delta\sin\theta + \sin\Delta\cos\theta\cos\varphi\right) = 0;$$
$$\varphi'' + \psi''\cos\theta - \psi'\theta'\sin\theta + \sin\Delta\sin\theta\sin\varphi = 0, \tag{9}$$

where prime denotes the derivative with respect to dimensionless time variable τ. First integrals (5)-(6) have the following dimensionless form

$$\hat I = \frac{1}{2}J\left(\theta'^2 + \psi'^2\sin^2\theta\right) + \frac{1}{2}\left(\varphi' + \psi'\cos\theta\right)^2 \tag{10}$$
$$+ \left(\cos\Delta\cos\theta - \sin\Delta\sin\theta\cos\varphi\right) = \text{const.}$$
$$\hat C = \left(J\sin^2\theta + \cos^2\theta\right)\psi' + \varphi'\cos\theta = \text{const.} \tag{11}$$

Since Lagrangian (1) of the system has only one cyclic coordinate, this model of hovering magnetic top has specific stationary solution which describes so-called synchronous motion [3]

$$\psi'(t) = \omega_0; \quad \theta(t) = \theta_0; \quad \varphi(t) = \varphi_0, \tag{12}$$

where ω_0, θ_0 and φ_0 are real constants. This solution satisfies Eq. (7) identically, while from Eq. (9) follows that the angle of rotation is $\varphi_0 = 0$. It will be show in the sequel that other possibilities which come from this equation ($\Delta = 0$ or $\theta_0 = 0$) are ruled out by stability conditions. Finally, Eq. (8) provides the following non-trivial constraint to the stationary solution (12)

$$\omega_0^2(J-1)\sin\theta_0\cos\theta_0 + \sin(\Delta + \theta_0) = 0. \tag{13}$$

From this equation, a stationary value of precessional angular velocity can be determined for given Δ and θ_0

$$\omega_0 = \sqrt{\frac{\sin(\Delta+\theta_0)}{(1-J)\sin\theta_0\cos\theta_0}}. \tag{14}$$

In real situations angles Δ and θ_0 are small and (14) can be approximated as

$$\omega_0 \approx \omega_{min}\sqrt{1 + \frac{\Delta}{\theta_0}}; \quad \omega_{min} = \sqrt{\frac{1}{1-J}}. \tag{15}$$

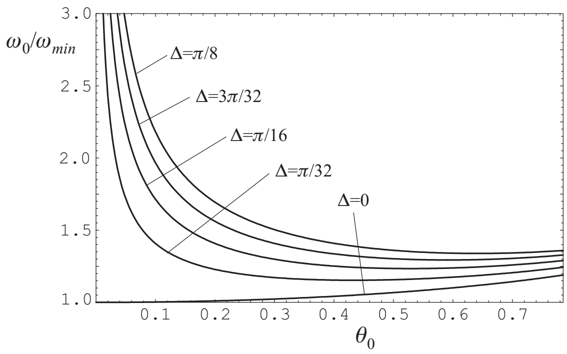

Figure 2. Stationary angular velocity ω_0 vs. ω_{min}.

The last result calls for brief explanation. The value ω_{min} was given by Flanders et al. (Eq. (8) in [3]) as a minimal value of precessional angular velocity needed for existence of synchronous stationary motion. This statement can be supported by the following arguments. For small values of the angles Δ and θ_0 equation (13) can be solved for θ_0

$$\theta_0 \approx \frac{\Delta}{\omega_0^2(1-J)-1}. \tag{16}$$

Since Δ is assumed constant and positive, and θ_0 is assumed positive, the following inequality has to be satisfied

$$\omega_0 > \sqrt{\frac{1}{1-J}} = \omega_{min}, \tag{17}$$

thus confirming the observation of [3] with constraint $J < 1$ (i.e. $J_\xi < J_\zeta$). Actually, ω_0 determined by (14) satisfies this inequality (see Fig. 2), except for $\Delta = \theta_0 = 0$, and (17) can be regarded as necessary condition for existence of stationary solution.

3. Variational equations

The first step in stability analysis is derivation of variational equations. They will be given in the form of the system of first-order of ordinary differential equations (ODE's). To that end the perturbations of stationary solution (12) have to be introduced

$$\begin{aligned}
&\psi'(t) = \omega_0 + x_1; \quad \theta(t) = \theta_0 + x_2; \quad \varphi(t) = x_3;\\
&\psi''(t) = x_1'; \quad \theta'(t) = x_4 = x_2'; \quad \varphi'(t) = x_5 = x_3';\\
&\theta''(t) = x_4'; \quad \varphi''(t) = x_5'.
\end{aligned} \tag{18}$$

By inserting (18) into (7)-(9) the following set of non-linear variational equations is obtained

$$\begin{aligned}
x_1' = \frac{1}{J}\{&\cot(\theta_0+x_2)\sin\Delta\sin x_3 - \cot(\theta_0+x_2)(\omega_0+x_1)x_4\\
&+(1-J)\csc^2(\theta_0+x_2)\sin(2(\theta_0+x_2))(\omega_0+x_1)x_4\\
&+\csc(\theta_0+x_2)x_4x_5\};
\end{aligned}$$

$$x_2' = x_4;$$

$$
\begin{aligned}
x_3' &= x_5;\\
x_4' &= \frac{1}{J}\{\cos(\theta_0+x_2)\cos x_3\,\sin\Delta+\sin(\theta_0+x_2)\cos\Delta\\
&\quad-(1-J)\cos(\theta_0+x_2)\sin(\theta_0+x_2)(\omega_0+x_1)^2\\
&\quad-\sin(\theta_0+x_2)(\omega_0+x_1)x_5\};\\
x_5' &= \frac{\csc^2(\theta_0+x_2)}{J}\left\{\left(\cos^2(\theta_0+x_2)+J\sin^2(\theta_0+x_2)\right)\right.\\
&\quad\times((\omega_0+x_1)x_4-\sin\Delta\sin x_3)\sin(\theta_0+x_2)\\
&\quad-((1-J)\sin(2(\theta_0+x_2))(\omega_0+x_1)\\
&\qquad\left.+\sin(\theta_0+x_2)x_5)\cos(\theta_0+x_2)x_4\right\}.
\end{aligned}
\tag{19}
$$

Linearized variational equations are obtained by expanding r.h.s. of (19) in the neighborhood of unperturbed state $x_i=0$, $i=1,\dots,5$

$$
\begin{aligned}
x_1' &= \frac{\cot\theta_0}{J}\left(x_3\sin\Delta+(1-2J)\omega_0x_4\right);\\
x_2' &= x_4;\\
x_3' &= x_5;\\
x_4' &= \frac{1}{J}\{-(1-J)\omega_0x_1\sin 2\theta_0\\
&\quad+\left(\cos(\Delta+\theta_0)-(1-J)\omega_0^2\cos 2\theta_0\right)x_2-\omega_0x_5\sin\theta_0\};\\
x_5' &= \frac{1}{J}\left\{-\frac{\sin\Delta}{\sin\theta_0}\left(\cos^2\theta_0+J\sin^2\theta_0\right)x_3\right.\\
&\quad\left.+\frac{\omega_0}{\sin\theta_0}\left(-(1-2J)\cos^2\theta_0+J\sin^2\theta_0\right)x_4\right\}.
\end{aligned}
\tag{20}
$$

The system (20) can be written in a compact form $\mathbf{x}'=\mathbf{A}\mathbf{x}$, where $\mathbf{x}=(x_1,\dots,x_5)^T$ is vector of perturbations and $\mathbf{A}$ is matrix of coefficients of the r.h.s. of variational equations (20).

4. Linear stability analysis

Linear stability analysis is based upon analysis of the eigenvalues of coefficient matrix $\mathbf{A}$, see Merkin [6] or Khalil [7]. If all the eigenvalues have negative real parts, unperturbed solution is asymptotically stable. If there is at least one eigenvalue with positive real part, unperturbed solution is unstable. Finally, if there are some eigenvalues with zero real part, while real parts of the other ones are negative, unperturbed solution is said to be marginally (neutrally) stable. Conclusion about stability in the first two cases does not depend on higher order terms in variational equations. However, in the case of marginal stability, linear stability analysis could not provide definitive answer: higher order terms could make the unperturbed solution neutrally stable, asymptotically stable or even unstable.

These limitations of linear stability analysis come on their own in the study of synchronous motion of hovering magnetic top. Namely, characteristic equation $\det(\mathbf{A}-\lambda\mathbf{I})=0$ is of the fifth degree, but can be reduced to the following special form

$$
\lambda\left(a\lambda^4+b\lambda^3+c\lambda^2+d\lambda+e\right)=0,
\tag{21}
$$

where

$$
\begin{aligned}
a &= 2J^3;\\
c &= J\{J(-2\cos\Delta\cos\theta_0) + (2 + J - J\cos 2\theta_0)\csc\theta_0\sin\Delta\\
&\quad +2\left(J^2 + (1-3J+2J^2)\cos^2\theta_0\right)\omega_0^2\};\\
e &= -\frac{1}{2}J\csc\theta_0\,\sin\Delta\{2\,(1+J+(1-J)\cos 2\theta_0)\cos(\Delta+\theta_0)\\
&\quad -(1-J)\,(2(1+J)\cos 2\theta_0 - (1-J)(-3+\cos 4\theta_0))\,\omega_0^2\},
\end{aligned}
\tag{22}
$$

while $b = d = 0$. Obviously, one eigenvalue is $\lambda = 0$ and marginal stability is the best one can expect from linear stability analysis.

In order to recover the results of Flanders et al. [3] it will be assumed that Δ and θ_0 are small. Furthermore, they are not independent, due to (16), and the following relation holds

$$
\frac{\Delta}{\theta_0} \approx \omega_0^2(1-J) - 1.
$$

Introducing this relation into (22), the approximate values of coefficients are obtained

$$
\begin{aligned}
a &= 2J^3;\\
c &= 2J\left(-2J + (1-2J+2J^2)\omega_0^2\right);\\
e &= 2J\left(1-(1-J)\omega_0^2\right)^2.
\end{aligned}
\tag{23}
$$

Non-zero eigenvalues are obtained as solutions of bi-quadratic equation $a\lambda^4 + c\lambda^2 + e = 0$. Solutions λ^2 ought to be real and negative in order to satisfy conditions of marginal stability. Since

$$
\lambda^2 = \frac{1}{2a}\left(-c \pm \sqrt{c^2 - 4ae}\right),
$$

coefficients of characteristic equation have to satisfy the following conditions

$$
c > 0; \quad c^2 - 4ae > 0; \quad -c \pm \sqrt{c^2-4ae} < 0 \Rightarrow 4ae > 0.
$$

The first two inequalities impose the following lower bounds for stationary angular velocity

$$
\omega_0 > \omega_1 = \left(\frac{2J}{1-2J+2J^2}\right)^{1/2}; \quad \omega_0 > \omega_2 = 2\sqrt{J},
\tag{24}
$$

while the third inequality is satisfied for any ω_0. For the purpose of comparison, inequality $(24)_2$ could be written in dimensional form

$$
\frac{mH}{\omega_0^2 J_\xi} < \frac{1}{4}\left(\frac{J_\zeta}{J_\xi}\right)^2,
$$

which was given in [3] as condition for existence of real roots. On the other hand, inequality $(24)_1$ have the following dimensional form

$$
\frac{mH}{\omega_0^2 J_\xi} < \frac{1}{2}\left(\frac{J_\zeta}{J_\xi}\right)^2 - \frac{J_\zeta}{J_\xi} + 1,
$$

which is a condition for existence of negative roots in [3].

Inequalities (24) are conditions of marginal linear stability and they have to be compared with necessary condition for existence of stationary solution (17). It can be shown that the following inequalities hold for $0 < J < 1$ (see Fig. 3)

$$
\omega_0 > \omega_{min} \geq \omega_2 \geq \omega_1.
$$

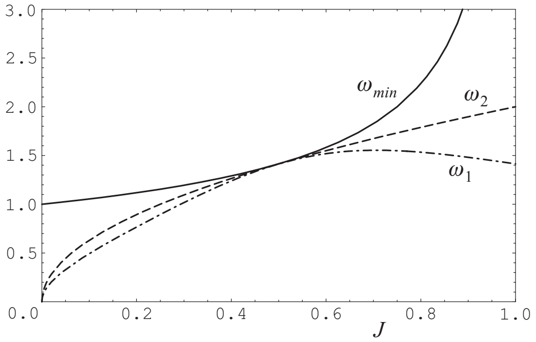

Figure 3. Stability bounds for angular velocity by linear theory.

In conclusion, stationary solution ω_0, determined by (14), is marginally stable in the sense of linear stability analysis. However, this conclusion cannot be regarded as a complete proof of stability of synchronous stationary motion for the reasons stated before.

5. Stability analysis by Lyapunov method

The problem encountered in this study is typical for systems in which some kind of energy conservation exists. As already mentioned, this model of hovering magnetic top has quadratic first integral (10), which is actually the (generalized) energy integral. It usually suffice for construction of Lyapunov function. Nevertheless, in some situations second order expansion of the energy integral, in the neighborhood of unperturbed solution, also contains linear terms, and thus does not fulfill the condition of definiteness. This is just the case in the present problem.

To resolve the problems of this kind Chetayev developed the procedure for construction of Lyapunov function using a combination of first integrals, if there exist more then one. The main problem consists in finding suitable combination of first integrals which is definite in the neighborhood of unperturbed solution. As a consequence, non-linear stability can be proved since the derivative of Lyapunov function, constructed in this way, is identically zero. This procedure is especially promising when the system posses linear first integral, apart from quadratic one.

In our problem, cyclic integral (11) can be adjoined to Jacobi's one (10) to construct the Lyapunov function in the following way

$$V = \hat{I} - \hat{I}_0 + \mu\left(\hat{C} - \hat{C}_0\right),$$

where $\hat{I}_0$ and $\hat{C}_0$ denote first integrals evaluated on unperturbed stationary solution (12). Introducing perturbations (18) into $\hat{I}$ and $\hat{C}$ in Lyapunov function one obtains

$$\begin{aligned} V = {} & \frac{1}{2}J\left(x_4^2 + (\omega_0 + x_1)^2 \sin^2(\theta_0 + x_2)\right) + \frac{1}{2}\left(x_5 + (\omega_0 + x_1)\cos(\theta_0 + x_2)\right)^2 \\ & + (\cos\Delta\cos(\theta_0 + x_2) - \sin\Delta\sin(\theta_0 + x_2)\cos x_3) \\ & - \frac{1}{2}J\omega_0^2 \sin^2\theta_0 - \frac{1}{2}\omega_0^2\cos^2\theta_0 - (\cos\Delta\cos\theta_0 - \sin\Delta\sin\theta_0) \qquad (25) \\ & + \mu\left\{\left(J\sin^2(\theta_0 + x_2) + \cos^2(\theta_0 + x_2)\right)(\omega_0 + x_1) + x_5\cos(\theta_0 + x_2)\right) \end{aligned}$$

$$- \left(J\sin^2\theta_0 + \cos^2\theta_0\right)\omega_0\big\}\,.$$

By expanding Lyapunov's function in Taylor series up to second order in the neighborhood of $x_i = 0$, $i = 1,\ldots,5$, it is find out that linear terms disappear when $\mu = -\omega_0$, either identically, or by means of relation (13). Thus, the following quadratic form is obtained

$$\begin{aligned} V = &\frac{1}{2}\left\{\left(J\sin^2\theta_0 + \cos^2\theta_0\right)x_1^2 + 2\cos\theta_0\, x_1 x_5 + x_5^2\right\} \\ &+ \frac{1}{2}\left((1-J)\omega_0^2\cos 2\theta_0 - \cos(\Delta+\theta_0)\right)x_2^2 \\ &+ \frac{1}{2}\sin\Delta\,\sin\theta_0 x_3^2 + \frac{1}{2}Jx_4^2 + O(\|\mathbf{x}\|^3). \end{aligned} \tag{26}$$

If this quadratic form is positive definite, there will exist a neighborhood of unperturbed solution $x_i = 0$ in which the Lyapunov function is positive definite. Consequently, according to Lyapunov's theorem, unperturbed solution will be stable with respect to small perturbations of initial conditions, since the derivative of (25) with respect to variational equations (19) is zero.

Definiteness of Lyapunov function will be checked in several steps. First, note that coefficients of x_3^2 and x_4^2 are positive (under the assumptions of the present study, i.e. smallness of Δ and θ_0). Second, quadratic form of x_1 and x_5 can be analyzed independently of the remaining terms. Its definiteness can be proved using Sylvester's criterion which says that matrix (quadratic form) is positive definite if an only if all the determinants associated with upper-left submatrices are positive. In our case the matrix reads

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{15} \\ c_{51} & c_{55} \end{pmatrix} = \begin{pmatrix} J\sin^2\theta_0 + \cos^2\theta_0 & \cos\theta_0 \\ \cos\theta_0 & 1 \end{pmatrix}$$

and Sylvester's criterion is reduced to the following inequalities

$$J\sin^2\theta_0 + \cos^2\theta_0 > 0; \quad J\sin^2\theta_0 > 0. \tag{27}$$

Note that second inequality imposes constraint $\theta_0 \neq 0$, which means that the axis of the top has to be slightly tilted with respect magnetic filed $\mathbf{H}$. It is in accordance with the existence of stationary value of angular velocity from Eq. (14).

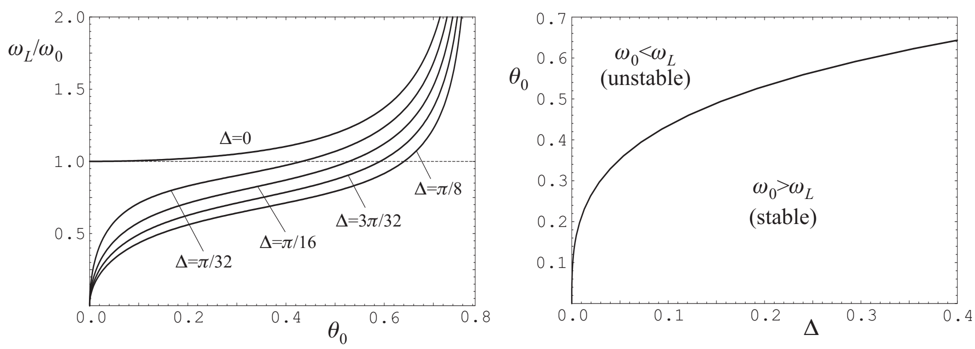

Figure 4. Stability bounds for angular velocity by Lyapunov method and critical curve in (Δ, θ_0) plane.

Finally, for the definiteness of (26) coefficient of x_2^2 has to be positive also. This leads to the following constraint for angular velocity

$$\omega_0 > \left(\frac{\cos(\Delta + \theta_0)}{(1-J)\cos 2\theta_0} \right)^{1/2} = \omega_L. \tag{28}$$

The value of ω_L has to be compared with ω_0 determined by (14). This provides a restriction on the values of Δ and θ_0 for which the stationary solution (12) is stable

$$\Delta > -\theta_0 + \arctan\left(\frac{1}{2}\tan 2\theta_0\right). \tag{29}$$

It is clear that for $\Delta = 0$ the conclusion about stability cannot be drawn, since the only solution is $\theta_0 = 0$. This is in complete agreement with real situation in which moment **m** of magnetic dipole deviates from the symmetry axis $O\zeta$ of the top.

These conclusions are illustrated in Fig. 4. Curves ω_L/ω_0, drawn for different values of Δ, show that that inequality (28) is satisfied only for a certain range of values of θ_0. Consequently, inequality (29) determines the critical curve in (Δ, θ_0) plane which divides it in two regions, stable and unstable, at least for small values of Δ and θ_0.

6. Conclusions

In this study we analyzed stability of synchronous stationary motion of hovering magnetic top. It was motivated by the fact that existing stability results were based upon conditions of marginal stability of linear stability analysis. The problem was treated in dimensionless form and principal results of previous studies were recovered and generalized. Main contribution of this study is non-linear stability analysis of the problem by means of Lyapunov method. A new result (29) came out from this analysis, determining the admissible values of the deviation angle Δ and nutation angle θ_0 for which the stationary synchronous motion is stable.

The motion of hovering magnetic top is analyzed under rather restrictive assumption that the center of mass is at rest. Taking into account its motion and analyzing the stability problem in its completeness is the problem for possible future studies.

Acknowledgement. The preparation of this paper is supported by the Ministry of Science of the Republic of Serbia within the project “Contemporary problems of mechanics of deformable bodies” (Project No. 144019).

References

[1] Simon M D, Heflinger L O and Ridgway S L (1997) Spin Stabilized Magnetic Levitation, *Am. J. Phys.*, **64**(4), pp. 286-292

[2] Berry M V (1996) The Levitron: an Adiabatic Trap for Spins, *Proc. R. Soc. Lond. A*, **452**, pp. 1207-1220

[3] Flanders P, Gov S, Shtrikman S and Thomas H (1999) On the Spinning Motion of Hovering Magnetic Top, *Physica D*, **126**, pp. 225-235.

[4] Dullin H R and Easton R W (1999) Stability of Levitrons, *Physica D*, **126**, pp. 1-17

[5] Krechetnikov R and Marsden J E (2006) On Destabilizing Effects of Two Fundamental Non-Conservative Forces, *Physica D*, **214**, pp. 25-32

[6] Merkin D R (1996) *Introduction to the Theory of Stability*, Springer-Verlag, Berlin.

[7] Khalil H K (2002) *Nonlinear systems*, Prentice-Hall, New Jersey.