

## HYPERBOLIC MULTI-TEMPERATURE MODEL FOR MIXTURES OF EULER FLUIDS

S. SIMIĆ

*Department of Applied Mechanics, Faculty of Technical Sciences  
University of Novi Sad, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia  
E-mail: ssimic@uns.ns.ac.yu*

In this review a macroscopic multi-temperature (MT) model for homogeneous mixtures of Euler fluids is proposed. Starting from usual set of governing equations a problem of modelling of homogeneous mixtures is analyzed and MT approach is motivated. Using the principles of extended thermodynamics and the theory of hyperbolic systems of balance laws the following questions are addressed: restriction on constitutive functions, hierarchical structure of hyperbolic subsystems and global existence of smooth solutions. As an application shock structure in binary non-reacting mixture is studied, as well as thermodynamic limit of MT model and different definitions of an average macroscopic temperature.

*Keywords:* Mixtures of fluids, extended thermodynamics.

### 1. Introduction

Starting from the model of homogeneous mixtures proposed by Truesdell [1], it is possible to develop several mathematical descriptions based upon different constitutive assumptions. Two main lines of modelling may be observed: one of them relies on the assumption that all the constituents have common — single temperature (ST), while another presumes that every constituent has its own temperature. Although the former assumption is widely adopted and agrees with experimental point of view, the latter is supported by subtle physical argumentation and has its roots both in continuum theories and kinetic theory of gases.

The aim of this study is to support the idea of introducing several temperatures in the theory of homogeneous mixtures. First, a process of modelling will be revisited and a new hyperbolic model will be proposed for mixtures of Euler fluids within the framework of extended thermodynamics. The second part of the review will be concerned with some applications

like the shock structure problem in binary mixtures, relation to classical thermodynamics (thermodynamic limit) and a question of appropriate definition of an average temperature of the mixture.

The results presented in this article are obtained in collaboration with Prof. Tommaso Ruggeri, University of Bologna, Italy. Their extensive exposition may be found in our recent papers [2–5].

## 2. Mathematical modelling of homogeneous mixtures

Theory of homogeneous mixtures is well-established within continuum thermomechanics, as well as in the framework of kinetic theory of gases. In either case basic macroscopic equations are conservation laws of mass, momentum and energy.

### 2.1. A route to multi-temperature model

In order to motivate the introduction of MT model in the mixture theory we shall give a sketch of the modelling process. To that end let us start with the simplest possible approach — model for the “whole mixture”

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) &= 0, \\ \frac{\partial(\rho \mathbf{v})}{\partial t} + \operatorname{div}(\rho \mathbf{v} \otimes \mathbf{v} - \mathbf{t}) &= \mathbf{0}, \\ \frac{\partial \left( \frac{1}{2} \rho v^2 + \rho \varepsilon \right)}{\partial t} + \operatorname{div} \left\{ \left( \frac{1}{2} \rho v^2 + \rho \varepsilon \right) \mathbf{v} - \mathbf{t} \mathbf{v} + \mathbf{q} \right\} &= 0. \end{aligned} \quad (1)$$

In this model macroscopic thermodynamic fields of density  $\rho$ , velocity  $\mathbf{v}$  and temperature  $T$ ,  $\mathbf{u} = (\rho, \mathbf{v}, T)$ , describe the mixture in the “average” sense. When the mixture is treated as neither viscous ( $\mathbf{t} = -p\mathbf{I}$ ,  $p$  being the pressure), nor heat-conducting ( $\mathbf{q} = \mathbf{0}$ ), continuum is regarded as Euler fluid and Eq. (1) is reduced to Euler’s equations of gas dynamics.

Although well-established, this model cannot cover important phenomena, like diffusion, which are typical for mixtures. To make description of these phenomena possible, the “structure” of the mixture have to be recognized, i.e.  $n$  constituents of the mixture ought to have an appropriate contribution to the mathematical model. This contribution is achieved through extension of the list of relevant state variables. This process could be motivated either by physical arguments, or by the need for certain mathematical structure of the model which appropriately correlates with them. As a result, different degrees of accuracy of physical description could be reached, as well as different mathematical levels of approximation.

Refinement of the mixture model could be performed in the following three steps:

- (1) introduction of partial densities  $\rho_\alpha$ ,  $\alpha = 1, \dots, n$ , and balance laws of masses for the constituents;
- (2) introduction of velocities of constituents  $\mathbf{v}_\alpha$  and corresponding balance laws of momenta;
- (3) introduction of temperatures of constituents  $T_\alpha$  and balance laws of energies.

The first step is typical for thermodynamics of irreversible processes (TIP) [6] and requires introduction of constitutive equations for diffusion fluxes even in the case of Euler fluids. The second step, made in the work of Müller [7], lies in the heart of extended thermodynamics of mixtures: by introducing balance laws for diffusion fluxes it removes the paradox of infinite speed of propagation of diffusion pulses predicted by classical Fick's law. The third step, although looking quite natural in this context, calls for sound physical or mathematical motivation. The idea of several temperatures, however odd it may look at first sight, is naturally imbedded into kinetic theory of gases [8,9]. It had been shown, both in kinetic and continuum approach, that MT assumption is quite reasonable when masses of constituents are disparate, like the masses of ions and electrons in plasmas [10,11].

The MT model of mixtures is finely tuned with the so-called metaphysical principles for mixtures established by Truesdell [1]. In accordance with these principles field variables for the "whole mixture", i.e.  $\rho$ ,  $\mathbf{v}$  and  $T$ , should be related to the fields describing the behaviour of constituents. Therefore, the following relations are established

$$\begin{aligned} \rho &= \sum_{\alpha=1}^n \rho_\alpha; \quad \mathbf{v} = \frac{1}{\rho} \sum_{\alpha=1}^n \rho_\alpha \mathbf{v}_\alpha; \quad \mathbf{u}_\alpha = \mathbf{v}_\alpha - \mathbf{v}; \quad \left( \sum_{\alpha=1}^n \rho_\alpha \mathbf{u}_\alpha = \mathbf{0} \right); \\ \varepsilon &= \varepsilon^I + \frac{1}{\rho} \sum_{\alpha=1}^n \frac{1}{2} \rho_\alpha u_\alpha^2; \quad \varepsilon^I = \frac{1}{\rho} \sum_{\alpha=1}^n \rho_\alpha \varepsilon_\alpha; \\ \mathbf{t} &= \sum_{\alpha=1}^n (\mathbf{t}_\alpha - \rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha); \quad \mathbf{q} = \sum_{\alpha=1}^n \left\{ \mathbf{q}_\alpha + \rho_\alpha \left( \varepsilon_\alpha + \frac{1}{2} u_\alpha^2 \right) \mathbf{u}_\alpha - \mathbf{t}_\alpha \mathbf{u}_\alpha \right\}. \end{aligned} \quad (2)$$

Although they suffice for derivation the MT model for mixtures, it has to be noted that in (2) there is no explicit relation whatsoever between temperatures of the constituents and the average macroscopic temperature of the mixture. Taking into account Eqs. (2) the set of balance laws for one

constituent, say  $n^{\text{th}}$  one, may be replaced by appropriate conservation laws (1) for the “whole mixture”, so that MT model for the mixture now reads

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) &= 0 \\
\frac{\partial(\rho \mathbf{v})}{\partial t} + \operatorname{div}(\rho \mathbf{v} \otimes \mathbf{v} - \mathbf{t}) &= \mathbf{0} \\
\frac{\partial \left( \frac{1}{2} \rho v^2 + \rho \varepsilon \right)}{\partial t} + \operatorname{div} \left\{ \left( \frac{1}{2} \rho v^2 + \rho \varepsilon \right) \mathbf{v} - \mathbf{t} \mathbf{v} + \mathbf{q} \right\} &= 0 \\
\frac{\partial \rho_b}{\partial t} + \operatorname{div}(\rho_b \mathbf{v}_b) &= \tau_b \\
\frac{\partial(\rho_b \mathbf{v}_b)}{\partial t} + \operatorname{div}(\rho_b \mathbf{v}_b \otimes \mathbf{v}_b - \mathbf{t}_b) &= \mathbf{m}_b \\
\frac{\partial \left( \frac{1}{2} \rho_b v_b^2 + \rho_b \varepsilon_b \right)}{\partial t} + \operatorname{div} \left\{ \left( \frac{1}{2} \rho_b v_b^2 + \rho_b \varepsilon_b \right) \mathbf{v}_b - \mathbf{t}_b \mathbf{v}_b + \mathbf{q}_b \right\} &= e_b
\end{aligned} \tag{3}$$

for  $b = 1, \dots, n-1$ . Source terms  $\tau_\alpha$ ,  $\mathbf{m}_\alpha$  and  $e_\alpha$  reflect mutual interactions of the constituents and satisfy the conditions

$$\sum_{\alpha=1}^n \tau_\alpha = 0; \quad \sum_{\alpha=1}^n \mathbf{m}_\alpha = \mathbf{0}; \quad \sum_{\alpha=1}^n e_\alpha = 0. \tag{4}$$

Analysis of this model is the main issue of this study.

## 2.2. Structure of the multi-temperature model

Several questions could be raised with regard to the structure of mathematical model — closure problem being one of the most important. The answers will be searched for within the framework of rational extended thermodynamics [12] by exploiting basic principles of this theory.

**Extended thermodynamics of MT mixtures.** The set of usual state variables  $\rho$ ,  $\mathbf{v}$  and  $T$  is extended with ones describing the behaviour of the constituents,  $\mathbf{u} = (\rho, \mathbf{v}, T, c_b, \mathbf{u}_b, \Theta_b)$ ,  $\Theta_b = T_n - T_b$ . Then, typically for extended thermodynamics, processes are described by balance laws (3)<sub>4–6</sub> implying the existence of source terms. Also, the local character of constitutive equations is imposed leading to the set of quasi-linear first order PDE's.

Two important restrictions have to be imposed to the model in this context. The first is invariance of balance laws with respect to Galilean transformations which reveals velocity dependence of source terms

$$\tau_b = \hat{\tau}_b; \quad \mathbf{m}_b = \hat{\tau}_b \mathbf{v} + \hat{\mathbf{m}}_b; \quad e_b = \hat{\tau}_b \frac{v^2}{2} + \hat{\mathbf{m}}_b \cdot \mathbf{v} + \hat{e}_b; \quad (5)$$

hat denotes terms which depend on objective quantities only. The second is compatibility with entropy inequality with convex entropy which determines the structure of source terms through residual inequality

$$\hat{\tau}_b = - \sum_{c=1}^{n-1} \varphi_{bc}(\mathbf{w}) \left( \frac{\mu_c - \frac{1}{2}u_c^2}{T_c} - \frac{\mu_n - \frac{1}{2}u_n^2}{T_n} \right); \quad (6)$$

$$\hat{\mathbf{m}}_b = - \sum_{c=1}^{n-1} \psi_{bc}(\mathbf{w}) \left( \frac{\mathbf{u}_c}{T_c} - \frac{\mathbf{u}_n}{T_n} \right); \quad \hat{e}_b = - \sum_{c=1}^{n-1} \theta_{bc}(\mathbf{w}) \left( -\frac{1}{T_c} + \frac{1}{T_n} \right),$$

where  $\varphi_{bc}(\mathbf{w})$ ,  $\psi_{bc}(\mathbf{w})$  and  $\theta_{bc}(\mathbf{w})$  are positive semi-definite matrix functions of objective quantities  $\mathbf{w}$  and  $\mu_c$  are chemical potentials of the constituents.

In the sequel our attention will be focused on the mixture of Euler fluids,  $\mathbf{t}_\alpha = -p_\alpha \mathbf{I}$ ,  $\mathbf{q}_\alpha = \mathbf{0}$ , where pressure of the mixture is determined by Dalton's law

$$p = \sum_{\alpha=1}^n p_\alpha. \quad (7)$$

**Hierarchy of hyperbolic subsystems.** Compatibility of the model with the entropy inequality is achieved by means of Lagrange multipliers [13] and construction of the main field  $\mathbf{u}' = (\Lambda^\rho, \Lambda^\mathbf{v}, \Lambda^\varepsilon, \Lambda^{\rho b}, \Lambda^{\mathbf{v} b}, \Lambda^{\varepsilon b})$ . Main field may be used for transformation of governing equations into symmetric hyperbolic form which reveals hierarchical—nested structure of hyperbolic subsystems. To that end, the most important components are the ones corresponding to non-equilibrium variables

$$\begin{aligned} \Lambda^{\rho b} &= \frac{1}{T_b} \left( \mu_b - \frac{1}{2}(\mathbf{u}_b + \mathbf{v})^2 \right) - \frac{1}{T_n} \left( \mu_n - \frac{1}{2}(\mathbf{u}_n + \mathbf{v})^2 \right); \\ \Lambda^{\mathbf{v} b} &= \frac{\mathbf{u}_b}{T_b} - \frac{\mathbf{u}_n}{T_n} - \left( \frac{1}{T_b} - \frac{1}{T_n} \right) \mathbf{v}; \\ \Lambda^{\varepsilon b} &= \frac{1}{T_n} - \frac{1}{T_b}. \end{aligned} \quad (8)$$

Assuming one or more main field components equal zero throughout the process, one may recognize the subset of governing equations which could be dropped leading to a principal subsystem [14]. The following results could be immediately obtained

- (i) if  $\Lambda^{\varepsilon_b} = 0$ , i.e. when MT assumption is dropped  $T_1 = \dots = T_n = T$ , energy balance laws could be discarded from the system and single-temperature model is obtained as a principal subsystem of a MT one;
- (ii) if the diffusion between constituents is also neglected,  $\mathbf{u}_b = \mathbf{0}$  leading to  $\Lambda^{\mathbf{v}_b} = 0$ , momentum balance laws may be discarded and principal subsystem characteristic for TIP is obtained.

If it is assumed that the mixture is consisted of non-reacting constituents,  $\tau_\alpha = 0$ , then preceding assumptions lead to an equilibrium subsystem. On the other hand, in the case of chemically reacting mixture conditions which lead to an equilibrium subsystem cannot be derived from Lagrangian multipliers in a straightforward manner, i.e. conditions  $\Lambda^{\rho_b} = 0$  have to be adjoined with the law of mass action. In the sequel only non-reacting mixtures will be analyzed.

**Qualitative analysis.** Recently a group of results appeared discussing global existence of smooth solutions [15,16] and stability of constant states [17]. These qualitative results are deeply based upon Shizuta-Kawashima condition (K-condition) of genuine coupling. It is worth to note that under certain assumptions K-condition may be violated in ST mixture models. On the other hand, it is satisfied in MT model without restrictions, providing another argument in favor of multi-temperature assumption.

### 3. Applications of hyperbolic multi-temperature model

Any theoretical model, however good it may be, need to be tested to prove its validity. Potential of the proposed hyperbolic model of MT mixtures (3) will be enlightened through the following three problems: shock structure in binary mixtures of Euler fluids, Maxwellian iteration and thermodynamic limit and discussion of average temperature of the mixture.

#### 3.1. Shock structure in binary mixtures of Euler fluids

The simplest possible way to test the mixture model is to analyze binary mixture. Governing equations (3) will be rearranged using the vector of diffusion flux  $\mathbf{J} = \rho_1 \mathbf{u}_1 = -\rho_2 \mathbf{u}_2$  and concentration variable  $c = \rho_1/\rho$ .

To close the system we shall assume that constituents are perfect gases which obey classical thermal and caloric equations of state  $p_\alpha = (k/m_\alpha)\rho_\alpha T_\alpha$ ,  $\varepsilon_\alpha = (k/m_\alpha)T_\alpha/(\gamma_\alpha - 1)$ . Furthermore, we shall assume that mixture as a whole could be described by constitutive functions of the same form, i.e.  $p = (k/m)\rho T$ ,  $\varepsilon_I = (k/m)T/(\gamma - 1)$ , where  $m$ ,  $T$  and  $\gamma$  are av-

erage atomic mass, average temperature and average ratio of specific heats of the mixture, respectively. In such a way  $m$ ,  $T$  and  $\gamma$  could be expressed from Eqs. (2) and (7)

$$T = c \frac{m}{m_1} T_1 + (1 - c) \frac{m}{m_2} T_2; \quad (9)$$

$$\frac{1}{m} = \frac{c}{m_1} + \frac{1 - c}{m_2}; \quad \frac{1}{\gamma - 1} = \frac{c}{\gamma_1 - 1} \frac{m}{m_1} \frac{T_1}{T} + \frac{1 - c}{\gamma_2 - 1} \frac{m}{m_2} \frac{T_2}{T}.$$

The set of of state variables is completed with the difference of temperatures of the constituents  $\Theta = T_2 - T_1$ , so the state vector reads  $\mathbf{u} = (\rho, \mathbf{v}, T, c, \mathbf{J}, \Theta)$ .

The structure of source terms is fixed by exploiting the results of kinetic theory [10]. Along with Eqs. (5)-(6) they provide the following relations

$$m_1 = \hat{m}_1 = -\frac{\rho_0 c_0 (1 - c_0) T_0}{\tau_J} \left( \frac{1}{c T_1} - \frac{1}{(1 - c) T_2} \right) \frac{J}{\rho}; \quad (10)$$

$$e_1 = \hat{m}_1 v + \hat{e}_1; \quad \hat{e}_1 = \frac{k}{m_1 (\gamma_1 - 1)} \frac{\rho_0 c_0 T_0^2}{\tau_T} \frac{\Theta}{T_1 T_2},$$

where  $\tau_J$  and  $\tau_T$  are relaxation times for diffusion and temperature related by

$$\tau_T = \frac{2}{3(\gamma_1 - 1)} \frac{m_1 + m_2}{(1 - c_0) m_1} \tau_J. \quad (11)$$

Interestingly enough, (11) implies that diffusion processes are attenuated faster than thermal non-equilibrium for a large class of mixtures.

Assuming the profile of the shock wave in the form of plane traveling wave  $\mathbf{u} = \mathbf{u}(\xi) = \mathbf{u}(x - st)$ ,  $\xi = x - st$  governing equations (3) are reduced to a set of ordinary differential equations. Shock structure then represents a heteroclinic orbit which asymptotically connects stationary points of the system. Problem is solved numerically for He-Ar and He-Xe mixtures for  $c_0 = 0.3$  and Mach number  $M_0 = 1.5$  in upstream equilibrium state.

Numerical solution, shown in Fig. 1, reveals that non-equilibrium profile of  $\Theta$  is wider than the profile of  $c$  and  $J$  which is in accordance with the ratio of relaxation times,  $(\tau_T/\tau_J)_{\text{He-Ar}} = 15.7$ ,  $(\tau_T/\tau_J)_{\text{He-Xe}} = 48.3$ . Average temperature is monotonically increasing function within the profile, whereas temperature of the heavier component has an overshoot—there is a region within a profile where its values are greater than downstream equilibrium temperature, the result typical for MT models. It can also be observed that greater mass difference implies stronger thermal imbalance so that it can be viewed as a main driving agent for temperature difference.

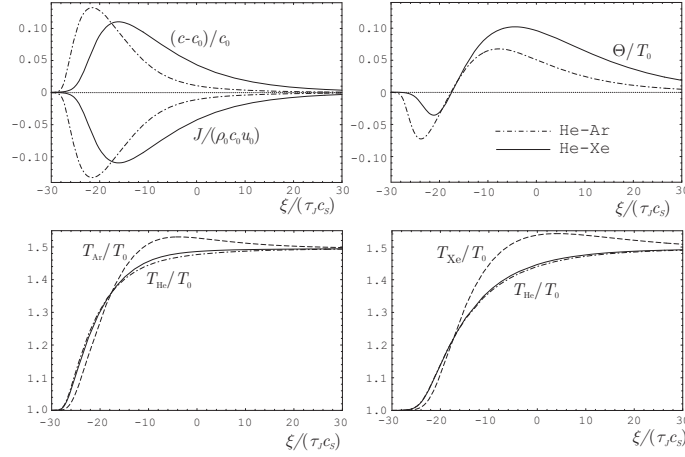


Fig. 1. Shock profiles in binary mixture:  $u = v - s$ ,  $c_S = \sqrt{\gamma_0(k/m_0)T_0}$  sound speed in upstream equilibrium state.

### 3.2. Thermodynamic limit of MT model

A natural question could be raised about relation of extended model (3) to classical non-local constitutive equations. In this survey the answer will be obtained through Maxwellian iteration—an iterative procedure which could be regarded as expansion in powers of relaxation times. In the case of binary mixture these results have been recently presented by Ruggeri & Simic. It was shown that first iterates for diffusion flux and temperature difference have the following form

$$\begin{aligned} \mathbf{J}^{(1)} &= L_{11} \text{grad} \left( \frac{\mu_2 - \mu_1}{T} \right) + L \text{grad} \left( \frac{1}{T} \right); \\ \Theta^{(1)} &= L_\theta (\gamma_1 - \gamma_2) \text{div} \mathbf{v}, \end{aligned} \quad (12)$$

where  $L_{11}$ ,  $L$  and  $L_\theta$  are phenomenological coefficients. Diffusion flux  $\mathbf{J}^{(1)}$  reproduces generalized Fick's law of diffusion, whereas  $\Theta^{(1)}$  represents entirely new result showing that temperature difference is proportional to  $\text{div} \mathbf{v}$  in the first iteration. It is a matter of direct calculation to generalize this conclusion to multi-component mixtures of Euler fluids.

### 3.3. Average temperature

The necessity for defining an average macroscopic temperature in MT model of mixtures comes from the lack of possibility for measuring the tempera-



tures of the constituents. Average temperature (9) has been defined with intention to express the constitutive equations for the mixture in the same form as for the constituents. However, this choice is not unique and other definitions of average temperature could also be put forward. One of the most striking motifs can be drawn from the analysis of homogeneous processes in binary mixture of Euler fluids. In this case spatial variations of state variables are neglected and governing equations are reduced to

$$\begin{aligned} \frac{d\rho_1}{dt} = 0; \quad \rho_1 \frac{d\mathbf{v}_1}{dt} = \mathbf{m}_1; \quad \frac{d}{dt} \left( \frac{1}{2} \rho_1 v_1^2 + \rho_1 \varepsilon_1 \right) &= e_1; \\ \frac{d\rho_2}{dt} = 0; \quad \rho_2 \frac{d\mathbf{v}_2}{dt} = -\mathbf{m}_1; \quad \frac{d}{dt} \left( \frac{1}{2} \rho_2 v_2^2 + \rho_2 \varepsilon_2 \right) &= -e_1. \end{aligned} \quad (13)$$

From Eqs. (13) one obtains conservation laws  $\rho_1 = \text{const.}$ ,  $\rho_2 = \text{const.}$  and  $\rho \mathbf{v} = \text{const.}$ , where  $\mathbf{v} = \mathbf{0}$  can be put without loss of generality due to Galilean invariance. Using equations of state for ideal gas, Eqs. (13)<sub>3,6</sub> lead to energy conservation equation

$$\frac{1}{2} \rho_1 v_1^2 + \frac{1}{2} \rho_2 v_2^2 + \frac{k}{m_1} \frac{\rho_1 T_1}{\gamma_1 - 1} + \frac{k}{m_2} \frac{\rho_2 T_2}{\gamma_2 - 1} = E = \text{const.}, \quad (14)$$

where  $E$  is evaluated for the following equilibrium conditions  $\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{0}$ ,  $T_1 = T_2 = T_0$

$$E = k \left( \frac{1}{m_1} \frac{\rho_1}{\gamma_1 - 1} + \frac{1}{m_2} \frac{\rho_2}{\gamma_2 - 1} \right) T_0. \quad (15)$$

Neglecting quadratic diffusive terms for processes not far from equilibrium, the following definition of average temperature could be derived from Eqs. (14)-(15)

$$T = T_0 = \frac{\rho_1 m_2 (\gamma_2 - 1) T_1 + \rho_2 m_1 (\gamma_1 - 1) T_2}{\rho_1 m_2 (\gamma_2 - 1) + \rho_2 m_1 (\gamma_1 - 1)}. \quad (16)$$

Two remarks have to be given. First, for processes near equilibrium, where higher order terms could be neglected, an average temperature (16) remains constant throughout the spatially homogeneous process. Second, this definition came from the energy equation, rather than from Dalton's law (7). In fact, it come from the assumption that intrinsic values of internal energy in MT and ST models are the same,  $\varepsilon_{MT}^I = \varepsilon_{ST}^I$ . This idea could be generalized to a definition of average temperature for multi-component mixtures

$$T = \frac{\sum_{\alpha=1}^n \frac{\rho_{\alpha}}{m_{\alpha}(\gamma_{\alpha}-1)} T_{\alpha}}{\sum_{\beta=1}^n \frac{\rho_{\beta}}{m_{\beta}(\gamma_{\beta}-1)}}. \quad (17)$$

This idea will be further developed in our forthcoming studies.

### Acknowledgment

This paper is dedicated to Professor Tommaso Ruggeri, teacher and friend. Its preparation was supported by the Ministry of Science of Serbia within the project *Contemporary Problems of Mechanics of Deformable Bodies*.

### References

1. C. Truesdell, *Rational Thermodynamics* (McGraw-Hill, New York, 1969).
2. T. Ruggeri and S. Simić, *Math. Methods Appl. Sci.* **30**, 827 (2007).
3. S. Simić and T. Ruggeri, Shock Structure in a Hyperbolic Mixture of Binary Mixture of Non-Reacting Gases, in *Proc. of 1st International Congress of Serbian Society of Mechanics*, (Kopanoik, Serbia, 2007).
4. T. Ruggeri and S. Simić, Mixture of Gases with Multi-temperature: Maxwellian Iteration, in *Asymptotic Methods in Non Linear Wave phenomena*, eds. T. Ruggeri and M. Sammartino (World Scientific, Singapore, 2007), pp.186-194.
5. T. Ruggeri and S. Simić, Mixture of Gases with Multi-temperature: Identification of a Macroscopic Average Temperature, *Memorie dell'Accademia delle Scienze, Lettere ed Arti di Napoli*, Liguori Editore, Napoli (*in press*).
6. S.R. de Groot and P. Mazur, *Non-Equilibrium Thermodynamics* (North-Holland, Amsterdam, 1962).
7. I. Müller, *Arch. Rational Mech Anal.* **28**, 1 (1968).
8. S.C. Chapman and T.G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, London, 1961).
9. J.M. Burgers, *Flow Equations for Composite Gases* (Academic Press, New York, 1969).
10. T.K. Bose, *High-temperature gas dynamics* (Springer-Verlag, Berlin, 2004).
11. Ya.B. Zel'dovich and Yu.P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena* (Dover Publications, Mineola, 2002).
12. I. Müller and T. Ruggeri, *Rational Extended Thermodynamics* (Springer-Verlag, New York, 1998).
13. I-S. Liu, *Arch. Rational Mech. Anal.* **46**, (1972).
14. G. Boillat and T. Ruggeri, *Arch. Rational Mech. Anal.* **137**, 305 (1997).
15. B. Hanouzet and R. Natalini, *Arch. Ration. Mech. Anal.* **163**, 89 (2003).
16. W.-A. Yong, *Arch. Ration. Mech. Anal.* **172**, 247 (2004).
17. T. Ruggeri and D. Serre, *Quart. Appl. Math* **62**, 163 (2004).