



## SHOCK STRUCTURE IN A HYPERBOLIC MODEL OF BINARY MIXTURE OF NON-REACTING GASES

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### Abstract

In this paper the problem of shock structure in binary gas mixture is studied with assumption that temperatures of the constituents may not be equal. Mathematical model is developed within the context of extended thermodynamics leading to a hyperbolic system of quasi-linear partial differential equations. Classical Euler's system of gas dynamics equations appears to be an equilibrium subsystem of the mixture system. Due to the presence of dissipative terms discontinuous shock-wave solution is smoothed out to a continuous shock structure (profile) connecting two equilibrium states. By assuming the shock profile in the form of plane traveling wave a set of ordinary differential equations is derived from the complete set of balance laws.

Numerical study revealed that there exists a smooth solution to the problem for shock speeds greater than the highest characteristic speed of the equilibrium subsystem. This solution confirms that mass difference is the main cause for the difference of temperatures of the constituents. Moreover, nonlinear form of source terms, obtained through the use of entropy principle, appear to be crucial for the problem in question since the intermediate states of the system are driven far from equilibrium.

**Key words:** hyperbolic systems, mixtures of gases, extended thermodynamics

### 1. Introduction

Since the first rational model of homogeneous mixtures, proposed by Truesdell [1] within the context of rational thermodynamics, two different approaches were developed: one with a single temperature (ST) field for the mixture, whereas another employed several temperature fields - one for each constituent. Both of them were carefully developed within the framework of continuum thermomechanics but with strikingly different consequences concerning the entropy principle and wave propagation (see e.g. Müller [2] and Müller & Ruggeri[3]). On the other hand, multi-temperature (MT) approach is naturally embedded into Maxwell's kinetic theory of mixtures as can be seen in Chapman & Cowling [4] and Burgers [5].

Although ST approach is intuitively likely to be accepted, the multi-temperature one comes on its own when atomic masses of the constituents are different. Its relevance is also supported by the analysis of plasma behavior at high temperatures performed by Kannappan & Bose [6]. Apart from physical reasons, these two theories appeared to be different in a mathematical sense also: MT systems do not admit the solution with common temperature  $T_1 = T_2 = \dots = T_n = T$ , even when this condition is initially imposed.

The aim of this study is to analyze the shock structure problem for binary mixture of non-reacting ideal gases. Propagation of non-linear waves, i.e. acceleration and shock waves in MT ideal fluid mixtures is already discussed by Bowen & Rankin [7] and Bowen & Chen [8] with conclusion that passage of non-linear waves drives apart the temperatures of constituents which have been initially equal. Analysis of shock structure in ST binary mixtures is performed by Sherman [9] using constitutive equations of Navier-Stokes and Fourier and the method proposed by Gilbarg & Paolucci [10], while MT model for binary and ternary mixtures was studied by Ruyev et al. [11].

Present analysis will be based upon a hyperbolic model for MT mixtures recently proposed by Ruggeri & Simić [12] within the context of extended thermodynamics. Classical Euler's system of gas dynamics equations appears to be an equilibrium subsystem of this MT system. It will be shown that smooth solution to the problem of shock structure exists for the shock speeds greater than the highest characteristic speed of the equilibrium subsystem. By numerical study the influence of mass difference and non-linear source terms will be revealed.

## 2. The multi-temperature model for a binary mixture

The model for simple homogenous mixtures is based upon the postulate that each constituent obeys the same balance laws as a single fluid, see [1]. They express the rate of change of mass momentum and energy

$$\begin{aligned} \partial_t \rho_\alpha + \operatorname{div}(\rho_\alpha \mathbf{v}_\alpha) &= \tau_\alpha; \\ \partial_t(\rho_\alpha \mathbf{v}_\alpha) + \operatorname{div}(\rho_\alpha \mathbf{v}_\alpha \otimes \mathbf{v}_\alpha - \mathbf{t}_\alpha) &= \mathbf{m}_\alpha; \\ \partial_t \left( \frac{1}{2} \rho_\alpha v_\alpha^2 + \rho_\alpha \varepsilon_\alpha \right) + \operatorname{div} \left\{ \left( \frac{1}{2} \rho_\alpha v_\alpha^2 + \rho_\alpha \varepsilon_\alpha \right) \mathbf{v}_\alpha - \mathbf{t}_\alpha \mathbf{v}_\alpha + \mathbf{q}_\alpha \right\} &= e_\alpha. \end{aligned} \quad (1)$$

For binary mixture we have  $\alpha = 1, 2$  with usual notation for field variables, while source terms are present due to mutual interaction of constituents. Since mass, momentum and energy of the whole mixture have to be conserved, source terms have to obey the following additional equations

$$\sum_\alpha \tau_\alpha = 0; \quad \sum_\alpha \mathbf{m}_\alpha = \mathbf{0}; \quad \sum_\alpha e_\alpha = 0. \quad (2)$$

Appropriate conservation laws for the mixture could be obtained through summation of corresponding balance of for the constituents

$$\begin{aligned} \partial_t \rho + \operatorname{div}(\rho \mathbf{v}) &= 0; \\ \partial_t(\rho \mathbf{v}) + \operatorname{div}(\rho \mathbf{v} \otimes \mathbf{v} - \mathbf{t}) &= \mathbf{0}; \\ \partial_t \left( \frac{1}{2} \rho v^2 + \rho \varepsilon \right) + \operatorname{div} \left\{ \left( \frac{1}{2} \rho v^2 + \rho \varepsilon \right) \mathbf{v} - \mathbf{t} \mathbf{v} + \mathbf{q} \right\} &= 0, \end{aligned} \quad (3)$$

where following variables have been introduced

$$\begin{aligned}
 \rho &= \sum_{\alpha} \rho_{\alpha} && \text{mass density;} \\
 \mathbf{v} &= \frac{1}{\rho} \sum_{\alpha} \rho_{\alpha} \mathbf{v}_{\alpha} && \text{mixture velocity;} \\
 \mathbf{u}_{\alpha} &= \mathbf{v}_{\alpha} - \mathbf{v}; \quad \left( \sum_{\alpha} \rho_{\alpha} \mathbf{u}_{\alpha} = \mathbf{0} \right) && \text{diffusion velocity;} \\
 \mathbf{t} &= \sum_{\alpha} (\mathbf{t}_{\alpha} - \rho_{\alpha} \mathbf{u}_{\alpha} \otimes \mathbf{u}_{\alpha}) && \text{stress tensor;} \\
 \varepsilon &= \frac{1}{\rho} \sum_{\alpha} \rho_{\alpha} \left( \varepsilon_{\alpha} + \frac{1}{2} u_{\alpha}^2 \right) && \text{internal energy;} \\
 \mathbf{q} &= \sum_{\alpha} \left\{ \mathbf{q}_{\alpha} + \rho_{\alpha} \left( \varepsilon_{\alpha} + \frac{1}{2} u_{\alpha}^2 \right) \mathbf{u}_{\alpha} - \mathbf{t}_{\alpha} \mathbf{u}_{\alpha} \right\} && \text{flux of internal energy.}
 \end{aligned} \tag{4}$$

The model that will be analyzed in the sequel will be consisted of conservation laws for the mixture (3) and balance laws (2) for one constituent, say  $\alpha = 1$ , with the following vector of state variables  $\mathbf{u} = (\rho, \mathbf{v}, \varepsilon, \rho_1, \mathbf{v}_1, \varepsilon_1)^T$ .

### 2.1. Restrictions to the structure of the model

It was shown in [12] that Galilean invariance of field equations and entropy principle impose additional restrictions to the structure of the model. Galilean invariance clarifies velocity dependence of densities, fluxes and source terms and results of this study can be summarized as through the following relations

$$\tau_{\alpha} = \hat{\tau}_{\alpha}; \quad \mathbf{m}_{\alpha} = \hat{\tau}_{\alpha} \mathbf{v} + \hat{\mathbf{m}}_{\alpha}; \quad e_{\alpha} = \frac{1}{2} \hat{\tau}_{\alpha} v^2 + \hat{\mathbf{m}}_{\alpha} \mathbf{v} + \hat{e}_{\alpha}. \tag{5}$$

where hat denotes terms which do not depend on mixture velocity  $\mathbf{v}$ .

Further restriction comes out from the assumption that constituents of the mixture are ideal gases which are neither viscous, nor heat-conducting

$$\mathbf{t}_{\alpha} = -p_{\alpha} \mathbf{I}; \quad \mathbf{q}_{\alpha} = \mathbf{0}, \tag{6}$$

which also restricts the structure of stress tensor and heat flux of the mixture (4)

$$\mathbf{t} = -p \mathbf{I} - \sum_{\alpha} (\rho_{\alpha} \mathbf{u}_{\alpha} \otimes \mathbf{u}_{\alpha}); \quad \mathbf{q} = \sum_{\alpha} \left\{ \rho_{\alpha} \left( \varepsilon_{\alpha} + \frac{1}{2} u_{\alpha}^2 \right) + p_{\alpha} \right\} \mathbf{u}_{\alpha}, \tag{7}$$

where  $p_{\alpha}$  and  $p = \sum_{\alpha} p_{\alpha}$  are partial and total pressures, respectively. Note that flux of internal energy of the mixture appears only due to diffusion. It will be assumed that constituents obey classical thermal and caloric equations of state

$$p_{\alpha} = \frac{k}{m_{\alpha}} \rho_{\alpha} T_{\alpha}; \quad \varepsilon_{\alpha} = \frac{p_{\alpha}}{\rho_{\alpha} (\gamma_{\alpha} - 1)}. \tag{8}$$

Final restriction comes from the compatibility of governing equations with supplementary balance law, i.e. entropy principle. It was shown [12] that this condition is satisfied if source terms have the following form in the case of binary mixture

$$\begin{aligned}\hat{\tau}_1 &= -\varphi_{11}(\mathbf{w}) \left( \frac{\mu_1 - u_1^2/2}{T_1} - \frac{\mu_2 - u_2^2/2}{T_2} \right); \\ \hat{\mathbf{m}}_1 &= -\psi_{11}(\mathbf{w}) \left( \frac{\mathbf{u}_1}{T_1} - \frac{\mathbf{u}_2}{T_2} \right); \quad \hat{e}_1 = -\theta_{11}(\mathbf{w}) \left( -\frac{1}{T_1} + \frac{1}{T_2} \right),\end{aligned}\tag{9}$$

where  $\mu_\alpha = \varepsilon_\alpha - T_\alpha S_\alpha + p_\alpha / \rho_\alpha$  are chemical potentials of the constituents, and  $\varphi_{11}(\mathbf{w})$ ,  $\psi_{11}(\mathbf{w})$  and  $\theta_{11}(\mathbf{w})$  are positive definite functions of objective quantities  $\mathbf{w}$  in the neighborhood of equilibrium state.

### 2.2. Transformation of variables

Governing equations could be transformed by means of change of variables recently introduced by Ruggeri [13]. Defining concentration  $c$  and diffusion flux vector  $\mathbf{J}$  as

$$c = \rho_1 / \rho; \quad \mathbf{J} = \rho_1 \mathbf{u}_1 = -\rho_2 \mathbf{u}_2 = \beta \mathbf{q},\tag{10}$$

where thermal inertia  $\beta = 1/(g_1 - g_2)$  and  $g_\alpha = \varepsilon_\alpha + p_\alpha / \rho_\alpha + u_\alpha^2/2$  are non-equilibrium enthalpies, the model can be written in the following form

$$\begin{aligned}\partial_t \rho + \operatorname{div}(\rho \mathbf{v}) &= 0; \\ \partial_t(\rho \mathbf{v}) + \operatorname{div}(\rho \mathbf{v} \otimes \mathbf{v} + p \mathbf{I} + \frac{\mathbf{J} \otimes \mathbf{J}}{\rho c(1-c)}) &= \mathbf{0}; \\ \partial_t \left( \frac{1}{2} \rho v^2 + \rho \varepsilon \right) + \operatorname{div} \left\{ \left( \frac{1}{2} \rho v^2 + \rho \varepsilon + p \right) \mathbf{v} + \left( \frac{\mathbf{v} \cdot \mathbf{J}}{\rho c(1-c)} + \frac{1}{\beta} \right) \mathbf{J} \right\} &= 0; \\ \partial_t(\rho c) + \operatorname{div}(\rho c \mathbf{v} + \mathbf{J}) &= \tau_1; \\ \partial_t(\rho c \mathbf{v} + \mathbf{J}) + \operatorname{div}(\rho c \mathbf{v} \otimes \mathbf{v} + \frac{\mathbf{J} \otimes \mathbf{J}}{\rho c} + \mathbf{v} \otimes \mathbf{J} + \mathbf{J} \otimes \mathbf{v} + \nu \mathbf{I}) &= \mathbf{m}_1; \\ \partial_t \left( \frac{1}{2} \rho c \left( \mathbf{v} + \frac{\mathbf{J}}{\rho c} \right)^2 + \rho c e \right) + \operatorname{div} \left\{ \left( \frac{1}{2} \rho c \left( \mathbf{v} + \frac{\mathbf{J}}{\rho c} \right)^2 + \rho c e + \nu \right) \left( \mathbf{v} + \frac{\mathbf{J}}{\rho c} \right) \right\} &= e_1.\end{aligned}\tag{11}$$

with  $\nu = p_1$  and  $e = \varepsilon_1$ . Note that (11)<sub>1,3</sub> are conservation laws for the mixture, while (11)<sub>4,6</sub> are balance laws for the first constituent. Thus, system (11) can be regarded as a model for a single fluid with structure.

### 2.3. Field variables and equilibrium subsystem

Total pressure  $p$  and intrinsic internal energy  $\varepsilon_I$ , defined as  $p = p_1 + p_2$  and  $\rho \varepsilon_I = \rho_1 \varepsilon_1 + \rho_2 \varepsilon_2$  could be expressed in the form of thermal and caloric equations of state for a single fluid

$$p = \frac{k}{m} \rho T; \quad \varepsilon_I = \frac{kT}{m(\gamma - 1)},\tag{12}$$

provided average atomic mass  $m$ , mixture temperature  $T$  and average ratio of specific heats  $\gamma$  are defined as follows

$$\begin{aligned} \frac{1}{m} &= \frac{c}{m_1} + \frac{1-c}{m_2}; \\ T &= c \frac{m}{m_1} T_1 + (1-c) \frac{m}{m_2} T_2; \\ \frac{1}{\gamma-1} &= \frac{c}{\gamma_1-1} \frac{m}{m_1} \frac{T_1}{T} + \frac{(1-c)}{\gamma_2-1} \frac{m}{m_2} \frac{T_2}{T}. \end{aligned} \quad (13)$$

For further analysis it will be useful to express the temperatures of the constituents in terms of mixture temperature  $T$  and temperature difference  $\Theta = T_2 - T_1$

$$T_1 = T - \frac{m}{m_2} (1-c) \Theta; \quad T_2 = T + \frac{m}{m_1} c \Theta. \quad (14)$$

Using these definitions the following set of field variables may chosen in this study  $\mathbf{u} = (\rho, \mathbf{v}, T, c, \mathbf{J}, \Theta)^T$ .

Equilibrium state of the mixture, denoted as  $\mathbf{u}_0$ , may be defined as a state without diffusion,  $\mathbf{J}_0 = \mathbf{0}$ , with uniform distribution of concentration,  $c = c_0 = \text{const.}$ , where the constituents have common temperature,  $\Theta_0 = 0$ . Density, velocity and temperature of the mixture are uniformly distributed,  $\rho = \rho_0 = \text{const.}$ ,  $\mathbf{v} = \mathbf{v}_0 = \text{const.}$ ,  $T = T_0 = \text{const.}$  Therefore, the system of conservation laws (11)<sub>1-3</sub>, rewritten for  $\mathbf{J} = \mathbf{0}$ , represents a classical Euler's system of gas dynamics equations and can be regarded as an equilibrium subsystem of the full system of governing equations (11). Definitions and properties of subsystems in the context of hyperbolic models one may find in paper of Boillat & Ruggeri [14].

Two remarks are ought to be given. Variables  $c$ ,  $\mathbf{J}$  and  $\Theta$  can be treated as non-equilibrium variables in the sense of extended thermodynamics. Therefore, they are primarily relevant for processes far from equilibrium state. Second remark is concerned with hyperbolicity. Since the equilibrium subsystem and the full system of governing equations are both hyperbolic, there appears a question about their characteristic speeds. For the full MT system they can be calculated from (1) and read

$$\lambda_{MT\alpha}^{(1)} = v_{\alpha n} - c_{s\alpha}; \quad \lambda_{MT\alpha}^{(2,3,4)} = v_{\alpha n}; \quad \lambda_{MT\alpha}^{(5)} = v_{\alpha n} + c_{s\alpha}, \quad (15)$$

where  $v_{\alpha n} = \mathbf{v}_\alpha \cdot \mathbf{n}$  are normal components of constituent velocities to the wave front and  $c_{s\alpha} = \sqrt{(k\gamma_\alpha/m_\alpha)T_\alpha}$  are local sound speeds for every constituent. For a ST equilibrium subsystem they read

$$\lambda_{ST}^{(1)} = v_n - c_{sE}; \quad \lambda_{ST}^{(2,3,4)} = v_n; \quad \lambda_{ST}^{(5)} = v_n + c_{sE}, \quad (16)$$

where  $v_n = \mathbf{v} \cdot \mathbf{n}$  and  $c_{sE} = \sqrt{(k\gamma/m)T}$  is sound speed of a single fluid in equilibrium conditions. It has been proven [14] that following sub-characteristic condition is valid

$$\min_{\alpha} (v_{\alpha n} - c_{s\alpha}^*) \leq \lambda_{ST}^{(1)}; \quad \max_{\alpha} (v_{\alpha n} + c_{s\alpha}^*) \geq \lambda_{ST}^{(5)}, \quad (17)$$

where  $c_{s\alpha}^* = \sqrt{(k\gamma_\alpha/m_\alpha)T}$  are sound speeds of the full system evaluated in equilibrium. It is very important to note that characteristic speeds of equilibrium subsystem are bounded by

characteristic speeds of the full system in equilibrium, but do not have to be the elements of the latter set.

#### 2.4. Explicit form of source terms

To close discussion about the model we have to give an explicit form of source terms (9). It can be provided either through theoretical study based upon kinetic theory or by matching with experimental data. Present analysis will be related to results of kinetic theory presented by Bose [15]. Firstly, it will be assumed that gases in the mixture are non-reacting, i.e.  $\tau_\alpha = 0$ . Next, if expressions  $\mathbf{m}_1$  and  $e_1$  are to be determined from kinetic theory, they have to be analyzed in linear form in the neighborhood of equilibrium state. By comparison with results presented in [15] we obtain

$$\psi_{11}(\mathbf{w}_E) = 2T_0 \frac{m_1 m_2}{m_1 + m_2} \Gamma'_{12}; \quad \theta_{11}(\mathbf{w}_E) = 3kT_0^2 \frac{m_1 m_2}{(m_1 + m_2)^2} \Gamma'_{12}, \quad (18)$$

where  $\Gamma'_{12}$  is volumetric collision frequency between gases. On the other hand, functions  $\psi_{11}$  and  $\theta_{11}$  could be related to relaxation times  $\tau_J$  and  $\tau_T$  for diffusion and temperature in equilibrium state

$$\psi_{11}(\mathbf{w}_E) = \frac{\rho_0 c_0 (1 - c_0) T_0}{\tau_J}; \quad \theta_{11}(\mathbf{w}_E) = \frac{k}{m_1 (\gamma_1 - 1)} \frac{\rho_0 c_0 T_0^2}{\tau_T}, \quad (19)$$

By comparison of equations (18) and (19) the following relation between relaxation times is obtained

$$\tau_T = \frac{2}{3(\gamma_1 - 1)} \frac{m_1 + m_2}{(1 - c_0) m_1} \tau_J. \quad (20)$$

Therefore, source terms (9) will be used in conjunction with equations (19) and (20) in numerical study of the shock structure.

### 3. Formulation of the shock structure problem

Shock structure is assumed to be a plane traveling wave which moves with a constant speed  $s$ . It will therefore be enough to analyze one-dimensional analogue of equations (9) which can be written in the following general form

$$\partial_t \mathbf{F}^0(\mathbf{u}) + \partial_x \mathbf{F}(\mathbf{u}) = \mathbf{f}(\mathbf{u}). \quad (21)$$

The solution will be assumed in the form  $\mathbf{u}(x - st) = \mathbf{u}(\xi)$ ,  $\xi = x - st$ , which reduces the original model (21) to the set of ordinary differential equations

$$\frac{d}{d\xi} \mathbf{F}(\mathbf{u}) = \mathbf{A}(\mathbf{u}) \frac{d\mathbf{u}}{d\xi} = \mathbf{f}(\mathbf{u}); \quad \mathbf{A}(\mathbf{u}) = \frac{\partial \mathbf{F}(\mathbf{u})}{\partial \mathbf{u}}. \quad (22)$$

Note that in equilibrium  $\mathbf{u}_0 = (\rho_0, v_0, T_0, c_0, 0, 0)^T$  vector of source terms vanishes,  $\mathbf{f}(\mathbf{u}_0) = \mathbf{0}$ . Therefore, equilibrium states of the mixture are stationary points of the ODE system (22). Important feature of stationary points is their non-uniqueness - apart from  $\mathbf{u}_0$  there appears family of stationary points  $\mathbf{u}_1(\mathbf{u}_0, s)$  which coincide with  $\mathbf{u}_0$  for certain values of  $s$ . It can be

shown that these stationary points are determined as solutions of Rankine-Hugoniot equations for the equilibrium subsystem, i.e. Euler's equations, and they tend to  $\mathbf{u}_0$  when  $s$  tends to its characteristic speed. This classical result becomes important here since it predicts the appearance of shock structure for speeds greater than the highest characteristic speed of equilibrium subsystem, rather than the full system. It is in accordance with our expectations that dissipation, here present through source terms, smears out the shock wave. Theoretical justification of this conclusion may be found in [16].

Analysis of stationary points is important for numerical solution of the problem. Since four equations in the system (22) are genuine conservation laws, four eigenvalues of the linearized system are equal zero. If stationary point  $\mathbf{u}_0$  corresponds to unperturbed equilibrium state in front of the shock and  $\mathbf{u}_1$  to perturbed equilibrium state behind the shock, then corresponding eigenvalues are real in the neighborhood of  $s = c_{sE}$  with the property that one of them changes the sign when wave speed crosses this value, while the other one remains negative. In terms of stability theory, stationary point  $\mathbf{u}_0$  loses its stability for  $s > c_{sE}$ , while  $\mathbf{u}_1$  becomes stable. Corresponding eigenvectors determine local behavior of state variables in the neighborhood of stationary points.

#### 4. Numerical solution and discussion

In order to make computational problem easier conservation laws of the system (22) can be exploited for elimination of certain state variables and reduction of order of the system. Careful study showed that it is more appropriate to use conservation laws of mass and momentum of the mixture and conservation law of mass for the constituent to express  $\rho$ ,  $J$  and  $T$  in terms of  $c$ ,  $v$  and  $\Theta$  and then solve the remaining three equations numerically. In such a way the appearance of singularities is avoided.

Numerical solution of the problem is found in dimensionless form by introduction of the following non-dimensional variables

$$\hat{\rho} = \frac{\rho}{\rho_0}; \quad \hat{u} = \frac{u}{c_{sE}}; \quad \hat{T} = \frac{T}{T_0}; \quad \hat{J} = \frac{J}{\rho_0 c_0 u_0}; \quad \hat{\Theta} = \frac{\Theta}{T_0}; \quad M = \frac{u_0}{c_{sE}}; \quad \hat{\xi} = \frac{\xi}{\tau_J c_{sE}}, \quad (23)$$

where  $u = v - s$  is relative velocity of the mixture with respect to wave of speed  $s$  and  $M$  is Mach number in equilibrium state. In such a way observer is attached to the shock wave with a mixture flowing from left (unperturbed equilibrium state for  $\xi \rightarrow -\infty$ ) to right (perturbed equilibrium state for  $\xi \rightarrow \infty$ ). Density  $\hat{\rho}$ , diffusion flux  $\hat{J}$  and temperature  $\hat{T}$  of the mixture can then be expressed in the form

$$\hat{\rho} = \frac{M}{\hat{u}}; \quad \hat{J} = 1 - \frac{c}{c_0}; \quad \hat{T} = \frac{m(c)}{m(c_0)} \frac{\hat{u}}{M} \left\{ 1 - \gamma M^2 \left[ \left( 1 + \frac{(c - c_0)^2}{c(1 - c)} \right) \frac{\hat{u}}{M} - 1 \right] \right\}. \quad (24)$$

Since unperturbed state have to be given, the following values are adopted

$$\hat{u}_0 = M = 1.5; \quad c_0 = 0.3; \quad \hat{T}_0 = 1.0; \quad \hat{\Theta}_0 = 0 \quad \Rightarrow \quad \hat{\rho}_0 = 1.0; \quad \hat{J}_0 = 0. \quad (25)$$

In this study two mixtures are studied, both of them being mixtures of monatomic inert gases, thus having the following values for ratios of specific heats  $\gamma_1 = \gamma_2 = 5/3$ . In particular, mixtures of He and Ar and He and Xe are analyzed. In both cases  $c$  denotes the concentration of lighter

gas, i.e. He. Ratios of atomic masses have the values  $(m_2/m_1)_{\text{He-Ar}} = 9.9805$  and  $(m_2/m_1)_{\text{He-Xe}} = 32.8037$ .

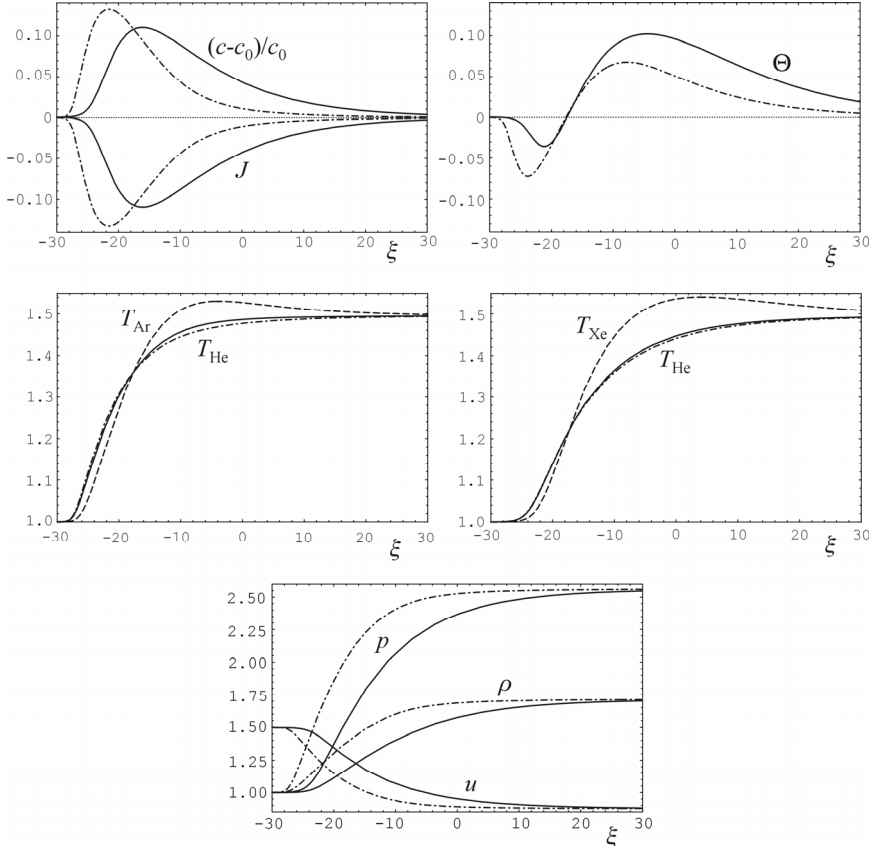


Fig. 1. Graphs of dimensionless state variables within a shock profile

Analysis of numerical solution shows that non-equilibrium variables, i.e.  $\hat{c}$ ,  $\hat{J}$  and  $\hat{\Theta}$  have the same equilibrium values in front and behind the shock, whereas within the profile their variations can be observed. That was expected also for the physical reasons: these variables are expected to be important only in processes which are far from equilibrium state. It can also be noted that in He-Ar mixture variation of concentration and diffusion flux, presented by dash-dot lines, have larger magnitude than variations of same variables in He-Xe mixture, presented by solid lines. On the other hand, maximum magnitude of temperature difference  $\hat{\Theta}$  is greater in He-Xe mixture than in He-Ar mixture. These two graphs show also that disturbances of concentration and diffusion flux within the same mixture are attenuated faster than disturbances of temperature difference. This is in accordance with the equation (20) which predicts that relaxation times for temperature are larger than relaxation times for diffusion, i.e.  $(\tau_T/\tau_J)_{\text{He-Ar}} = 15.7$  and  $(\tau_T/\tau_J)_{\text{He-Xe}} = 48.3$ .



Graphs of temperatures within the profile are crucial for the present study. Namely, it can be observed that in both cases temperature of the mixture (solid line) is monotonically increasing function. At the same time the temperature of heavier gas have an overshoot – there is a region in which it is greater than the equilibrium value behind the shock. It is also obvious that larger mass difference between the constituents produces larger temperature difference between them and mixture temperature as well. Therefore, one of initial assumptions of the study is confirmed – for mixtures whose constituents have disparate masses MT approach is appropriate.

Although the same results are deduced in [11] they were based upon Navier-Stokes-Fourier model, which is essentially parabolic. Here, mild assumptions about dissipative mechanism, i.e. diffusion and temperature difference, led to a hyperbolic model for the mixture and strikingly similar qualitative description of the shock structure. It is important to note that non-linear structure of source terms have been crucial to obtain physically appropriate results. Same analysis with source terms linearized around equilibrium state gave valid results only for very weak shocks and showed large discrepancy with non-linear theory even with data used in this study.

Finally, last graph presents the structure of equilibrium variables  $\hat{u}$ ,  $\hat{\rho}$  and  $\hat{p}$  within the profile for He-Ar (dash-dot) and He-Xe (solid) mixture.

Few remarks have to added to this discussion. In numerical analysis the value of Mach numbers for both mixtures was adopted to be  $M = 1.5$ . Although it could be criticized as too small, there is a reason for such a choice. Namely, in hyperbolic dissipative models equipped with convex entropy continuous shock structure ceases to exist when shock speed exceeds the highest characteristic speed of the full system, see Boillat & Ruggeri [17] for the proof. These speeds are characterized by the corresponding critical values  $M^*$  of Mach numbers, here being  $M_{\text{He-Ar}}^* = 1.64$  and  $M_{\text{He-Xe}}^* = 1.76$ . Therefore, in order to obtain continuous solution of the shock structure problem (22) one have to choose  $M < M^*$ . It can also be shown that critical value of Mach number also depends on the equilibrium concentration – it is monotonically decreasing function of  $c$ .

To extend the region of validity of the model, and obtain more accurate description of the problem, other dissipative mechanisms like viscosity and heat conduction have to be taken into account. Within the context of extended thermodynamics these models are referred to as 13 moment models. Recently, Barbera & Valenti [18] studied the model of binary mixture relied on 13 moment approach and studied global existence of solutions via Kawashima condition. The question of shock structure remained open in this context.

Vast field of study remained out of the scope of this paper due to assumption that there is no chemical reaction whatsoever,  $\tau_\alpha = 0$ . Problems which arise when this assumption is dropped are supposed to be themes for prospective studies.

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