MIXTURE OF GASES WITH MULTI-TEMPERATURE: MAXWELLIAN ITERATION

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In this paper a hyperbolic model is proposed for mixtures of gases which are neither viscous, nor heat-conducting (Eulerian fluids). It is built upon assumption that each constituent obeys it's own temperature. Restrictions to the structure of the model come out from basic principles of extended thermodynamics, i.e. Galilean invariance of balance laws and entropy inequality. Hierarchy of hyperbolic subsystems is recognized, with a single-temperature model as principal subsystem and classical Euler's equations as equilibrium subsystem. Finally, in order to relate this model to classical thermodynamics, a Maxwellian iteration is performed in the case of binary mixture, giving rise to a relation between the difference of non-equilibrium temperatures of constituents and classical fields.

Keywords: mixture of fluids, extended thermodynamics.

1. Introduction

Accurate description of non-equilibrium phenomena is an intricate problem which calls for permanent improvement of mathematical models and thorough analysis of constitutive assumptions. An example is the mixture theory of gases. In classical thermodynamics of irreversible phenomena (TIP) n–component mixture had been described by means of $n + 4$ fields of mass densities ρ_α, $\alpha = 1, \ldots, n$, the mixture velocity $\mathbf{v}$ and the mixture temper-

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ature T. On the other hand, within the framework of rational thermodynamics Truesdell[1] proposed a model of homogeneous mixtures. The basis of this approach is expressed as a principle that all properties of the mixture must be mathematical consequences of properties of the constituents. In accordance with this approach Müller[2] developed appropriate constitutive theory for the mixtures. This model described mixture using $4n+1$ field of densities ρ_α, constituent velocities $\mathbf{v}_\alpha$ and mixture temperature T.

In parallel to the single–temperature (ST) model, a multi–temperature (MT) one was developed within the same context[1], albeit independently of the former. The MT model was described with $5n$ fields of densities, velocities and temperatures of the constituents $(\rho_\alpha, \mathbf{v}_\alpha, T_\alpha)$. The differences between these two models were apparent, in particular with respect to entropy principle and wave propagation, see Refs. 2,3 and references cited therein. Moreover, two theories are quite different also in mathematical sense — their solutions cannot be related in a simple way since MT model, in principle, does not admit a solution of the form $T_1 = \ldots = T_n = T$, even when the initial temperatures of the constituents are equal. Although MT approach could be criticized for physical reasons, it is naturally imbedded into Maxwell's kinetic theory of mixtures[4,5].

This study has an intention to relate these two models using the basic principles of extended thermodynamics[3] and hierarchical structure of hyperbolic subsystems, as explained by Boillat and Ruggeri[6]. These results will facilitate the study of global existence and asymptotic behavior of the solutions. Finally, through the model of binary mixture a relation between the present model and the classical one will be established through the application of Maxwellian iteration procedure. Extensive explanation of results presented in Sects. 2 and 3 can be found in Ref. 7.

2. The multi–temperature model

Following the postulate of rational thermodynamics[1] that each constituent obeys the same balance laws as a single fluid, $5n$ balance laws of mass, momentum and energy for constituents could be used as governing equations for the MT mixture, with appropriate production terms due to mutual interaction of the constituents. However, it is convenient and conceptually adequate to write down also conservation laws of mass, momentum and energy of the whole mixture. As a consequence mathematical model for the MT mixture could be consisted of 5 conservation laws for the mixture as a whole and $5(n-1)$ balance laws for $n-1$ constituents, dropping the balance

laws for say n^{th} constituent

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{v}) = 0;$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \mathrm{div}(\rho \mathbf{v} \otimes \mathbf{v} - \mathbf{t}) = \mathbf{0};$$

$$\frac{\partial \left(\frac{1}{2} \rho v^2 + \rho \varepsilon \right)}{\partial t} + \mathrm{div} \left\{ \left(\frac{1}{2} \rho v^2 + \rho \varepsilon \right) \mathbf{v} - \mathbf{tv} + \mathbf{q} \right\} = 0;$$

$$\frac{\partial \rho_b}{\partial t} + \mathrm{div}(\rho_b \mathbf{v}_b) = \tau_b \qquad\qquad (1)$$

$$\frac{\partial (\rho_b \mathbf{v}_b)}{\partial t} + \mathrm{div}(\rho_b \mathbf{v}_b \otimes \mathbf{v}_b - \mathbf{t}_b) = \mathbf{m}_b, \qquad (b = 1, 2, \ldots, n-1)$$

$$\frac{\partial \left(\frac{1}{2} \rho_b v_b^2 + \rho_b \varepsilon_b \right)}{\partial t} + \mathrm{div} \left\{ \left(\frac{1}{2} \rho_b v_b^2 + \rho_b \varepsilon_b \right) \mathbf{v}_b - \mathbf{t}_b \mathbf{v}_b + \mathbf{q}_b \right\} = e_b,$$

Here usual notation of continuum thermomechanics has been exploited. Density ρ, (barycentric) velocity $\mathbf{v}$ and internal energy density ε of the mixture, as well as stress tensor $\mathbf{t}$ and flux of internal energy $\mathbf{q}$ are defined as

$$\rho = \sum_{\alpha=1}^{n} \rho_\alpha; \quad \mathbf{v} = \frac{1}{\rho} \sum_{\alpha=1}^{n} \rho_\alpha \mathbf{v}_\alpha; \quad \varepsilon = \frac{1}{\rho} \sum_{\alpha=1}^{n} \rho_\alpha \left(\varepsilon_\alpha + \frac{1}{2} u_\alpha^2 \right);$$

$$\mathbf{t} = \sum_{\alpha=1}^{n} (\mathbf{t}_\alpha - \rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha); \qquad\qquad (2)$$

$$\mathbf{q} = \sum_{\alpha=1}^{n} \left\{ \mathbf{q}_\alpha + \rho_\alpha \left(\varepsilon_\alpha + \frac{1}{2} u_\alpha^2 \right) \mathbf{u}_\alpha - \mathbf{t}_\alpha \mathbf{u}_\alpha \right\}.$$

In (2) $\mathbf{u}_\alpha = \mathbf{v}_\alpha - \mathbf{v}$ denotes diffusion velocity with obvious property $\sum_{\alpha=1}^{n} \rho_\alpha \mathbf{u}_\alpha = \mathbf{0}$ and production terms satisfy following relations

$$\sum_{\alpha=1}^{n} \tau_\alpha = 0; \quad \sum_{\alpha=1}^{n} \mathbf{m}_\alpha = \mathbf{0}; \quad \sum_{\alpha=1}^{n} e_\alpha = 0. \qquad (3)$$

Equations (1) are the governing equations for thermodynamic processes in MT mixtures. The goal of extended thermodynamics is to determine the $5n$ fields describing the state of the mixture as a whole and the states of $n-1$ constituents. The state of nth constituent can be obtained via $(2)_{1-3}$ and constitutive equations.

2.1. *Euler fluids*

So far nothing has been told about constitutive assumptions and the structure of production terms. Firstly, our attention will be restricted to so-called

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Euler fluids — fluids which are neither viscous, nor heat–conducting. This assumption leads to the following equations for stress tensors and heat fluxes

$$\mathbf{t}_\alpha = -p_\alpha \mathbf{I}; \quad \mathbf{q}_\alpha = \mathbf{0}; \quad (\alpha = 1, \ldots, n), \tag{4}$$

where p_α denote partial pressures.

Furthermore, it will be assumed that constituents obey classical thermal and caloric equations of state

$$p_\alpha = \frac{k}{m_\alpha} \rho_\alpha T_\alpha; \quad \varepsilon_\alpha = \frac{p_\alpha}{\rho_\alpha(\gamma_\alpha - 1)}; \quad (\alpha = 1, \ldots, n), \tag{5}$$

k being the Boltzmann constant and m_α is the atomic mass of the α-constituent.

2.2. *Galilean invariance*

The MT model for the mixture of Euler fluids are a particular example of the system of balance laws

$$\partial_t \mathbf{F}^0 + \partial_i (\mathbf{F}^0 v^i + \mathbf{G}^i) = \mathbf{F}, \tag{6}$$

where $\mathbf{F}^0$, $\mathbf{G}^i$ and $\mathbf{F}$ are densities, non-convective fluxes and production terms, respectively. Principle of relativity requires that equations (6) are invariant with respect to Galilean transformations. As a consequence, see Ruggeri[8], there exists a linear operator $\mathbf{X}(\mathbf{v})$ such that:

$$\begin{aligned}
\mathbf{F}^0(\mathbf{w}, \mathbf{v}) &= \mathbf{X}(\mathbf{v})\hat{\mathbf{F}}^0(\mathbf{w}), \\
\mathbf{G}^i(\mathbf{w}, \mathbf{v}) &= \mathbf{X}(\mathbf{v})\hat{\mathbf{G}}^i(\mathbf{w}), \\
\mathbf{F}(\mathbf{w}, \mathbf{v}) &= \mathbf{X}(\mathbf{v})\hat{\mathbf{F}}(\mathbf{w}),
\end{aligned} \tag{7}$$

where $\mathbf{v}$ is the mixture velocity and $\mathbf{w}$ is a vector of the other field variables and the hat indicates, also in the follows, the corresponding quantity evaluated at zero velocity. These conditions reveal the velocity dependence of production terms in the following way[7]

$$\begin{aligned}
\tau_b &= \hat{\tau}_b; \\
\mathbf{m}_b &= \hat{\tau}_b \mathbf{v} + \hat{\mathbf{m}}_b; \quad (b = 1, \ldots, n-1) \\
e_b &= \hat{\tau}_b \frac{v^2}{2} + \hat{\mathbf{m}}_b \cdot \mathbf{v} + \hat{e}_b,
\end{aligned} \tag{8}$$

while production terms for n^{th} constituent can be derived from Eq. (3).

2.3. *Entropy principle and its restrictions*

Apart from Galilean invariance entropy inequality, i.e. supplementary balance law

$$\partial_t h^0 + \partial_i(h^0 v^i + \varphi^i) = \Sigma \leq 0, \qquad (9)$$

imposes another important restriction to the structure of constitutive functions, in particular production terms. Except for the sign h^0, φ^i and Σ are respectively the entropy density, the entropy flux and the entropy production. Since balance laws and entropy inequality are both quasi-linear equations, their compatibility can be achieved through the use of the main field[9,10] $\mathbf{u}'$. Denoting the main field components by

$$\mathbf{u}' = \frac{\partial h^0}{\partial \mathbf{u}} = (\Lambda^\rho, \boldsymbol{\Lambda}^{\mathbf{v}}, \Lambda^\varepsilon, \Lambda^{\rho_b}, \boldsymbol{\Lambda}^{\mathbf{v}_b}, \Lambda^{\varepsilon_b}); \quad (b = 1, \ldots, n-1), \qquad (10)$$

and exploiting the so-called residual inequality

$$\Sigma = \mathbf{u}' \cdot \mathbf{F} = \hat{\mathbf{u}}' \cdot \hat{\mathbf{F}} = \sum_{b=1}^{n-1} \left(\hat{\Lambda}^{\rho_b} \hat{\tau}_b + \hat{\boldsymbol{\Lambda}}^{\mathbf{v}_b} \cdot \hat{\mathbf{m}}_b + \hat{\Lambda}^{\varepsilon_b} \hat{e}_b \right) \leq 0, \qquad (11)$$

one may obtain more explicit structure of internal parts of production terms. Choosing them in a way that (11) becomes a quadratic form, the following expressions are obtained[7]

$$\hat{\tau}_b = -\sum_{c=1}^{n-1} \varphi_{bc}(\mathbf{w}) \left(\frac{\mu_c - \frac{1}{2}u_c^2}{T_c} - \frac{\mu_n - \frac{1}{2}u_n^2}{T_n} \right); \qquad (12)$$

$$\hat{\mathbf{m}}_b = -\sum_{c=1}^{n-1} \psi_{bc}(\mathbf{w}) \left(\frac{\mathbf{u}_c}{T_c} - \frac{\mathbf{u}_n}{T_n} \right); \quad \hat{e}_b = -\sum_{c=1}^{n-1} \theta_{bc}(\mathbf{w}) \left(-\frac{1}{T_c} + \frac{1}{T_n} \right),$$

where $\mu_\alpha = \varepsilon_\alpha - T_\alpha S_\alpha + p_\alpha/\rho_\alpha$, $\alpha = 1, \ldots, n$ are chemical potentials of the constituents and $\varphi_{bc}(\mathbf{w})$, $\psi_{bc}(\mathbf{w})$ and $\theta_{bc}(\mathbf{w})$, $b, c = 1, \ldots, n-1$ are positive definite matrix functions of the objective quantities that we assume symmetric according with Onsager idea.

3. Principal subsystems and asymptotic behavior

Main field (10) can be used to transform the system of balance laws (6) into symmetric hyperbolic form

$$\partial_t \left(\frac{\partial h'^0}{\partial \mathbf{u}'} \right) + \partial_i \left(\frac{\partial h'^i}{\partial \mathbf{u}'} \right) = \mathbf{F}. \qquad (13)$$

We split into two blocks of R^M and R^{N-M} vectors the main field and the production R^N-vectors: $\mathbf{u}' \equiv (\mathbf{v}', \mathbf{w}')$, $\mathbf{F} \equiv (\mathbf{f}(\mathbf{u}'), \mathbf{g}(\mathbf{u}'))$.

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Assigned a constant value $\mathbf{w}'_*$ to the second group of variables, the system of the first M equations

$$\partial_t \left(\frac{\partial h'^0(\mathbf{v}', \mathbf{w}'_*)}{\partial \mathbf{v}'} \right) + \partial_i \left(\frac{\partial h'^i(\mathbf{v}', \mathbf{w}'_*)}{\partial \mathbf{v}'} \right) = \mathbf{f}(\mathbf{v}', \mathbf{w}'_*) \tag{14}$$

is called[6] a *principal subsystem* of (13).

It is also symmetric hyperbolic and obey the sub-characteristic condition and sub-entropy law[6]. A special class of systems (13) is the one for which $\mathbf{f}(\mathbf{u}') \equiv \mathbf{0}$. In this case, for $\mathbf{w}'_* = \mathbf{0}$ system (14) appears to be an equilibrium subsystem in which entropy production $-\Sigma|_E$ vanishes and attains its minimum.

3.1. *Principal subsystems in MT mixture model*

In the case of MT mixture an appropriate hierarchy of subsystems could be obtained using procedure described above. Most interesting cases are listed below[7]:

Case 1. A single-temperature model is a principal subsystem of the MT model. When $\Lambda^{\varepsilon_b} = 0$, $b = 1, \ldots, n-1$ is put, one obtains

$$T_1 = \ldots = T_n = T, \tag{15}$$

and balance laws for the energy should be dropped from the model. At the same time all the constitutive functions have to be evaluated in accordance with this restriction.

Case 2. If along with $\Lambda^{\varepsilon_b} = 0$, $\mathbf{\Lambda}^{\mathbf{v}_b} = 0$, $b = 1, \ldots, n-1$ is valid, the following restriction is obtained

$$\mathbf{u}_1 = \ldots = \mathbf{u}_n = \mathbf{0}, \tag{16}$$

which leads to a single temperature model where balance laws for momenta have to be dropped. This kind of model is widely accepted for modeling of detonation phenomena.

Case 3. If yet another condition is imposed Λ^{ρ_b}, $b = 1, \ldots, n-1$, the following restriction is obtained

$$\mu_1 = \ldots = \mu_n = \mu, \tag{17}$$

giving rise to equilibrium densities ρ_α of the constituents. Since balance laws for the masses have to be dropped, one arrives to the classical Euler's system of equations of gas dynamics, here representing an equilibrium subsystem.

3.2. *Qualitative analysis*

The MT model for mixtures is a particular case of a system of balance laws (6) which is dissipative due to the presence of the productions that satisfy the entropy principle. On the other hand the model is of mixed type since five equations are conservation laws while the rest of the system is consisted of true balance laws

$$\mathbf{F}(\mathbf{u}) = \begin{pmatrix} 0 \\ \mathbf{g}(\mathbf{u}) \end{pmatrix}.$$

For this class of systems the coupling condition of Shizuta and Kawashima[11] (K-condition) can ensure the global existence of smooth solutions, meaning that dissipation in balance laws prevails the hyperbolicity of conservation laws. Actually, the K-condition reads: *In the equilibrium manifold any characteristic eigenvector is not in the null space of* $\nabla \mathbf{F}$, i.e.

$$\nabla \mathbf{F} \cdot \mathbf{d}^{(i)}\Big|_{E} \neq 0 \quad \forall i = 1, \dots, N. \tag{18}$$

where $\mathbf{d}^{(i)}$ are the right-eigenvectors of the hyperbolic system (6):

$$(\mathbf{A}_n - \lambda \mathbf{I}) \, \mathbf{d} = 0, \tag{19}$$

where $\mathbf{A}_n = \mathbf{A}^i n_i$, $\mathbf{A}^i = \nabla \mathbf{F}^i$, $\mathbf{u} = \mathbf{F}^0$, $\nabla = \partial/\partial \mathbf{u}$ and E stands for the equilibrium state.

It has been proven recently that if the system of balance laws (6) is endowed with a convex entropy law, and it is dissipative, then the K-condition becomes a sufficient condition for the existence of global smooth solutions provided that the initial data are sufficiently smooth.[12,13] Furthermore, Ruggeri and Serre[14] proved the stability of constant states in one-dimensional case.

After an extensive calculation[7] it has been shown that K–condition is satisfied for the MT model of mixtures and that, according to above mentioned results, the following statement is valid:

If the initial data of the MT model are perturbations of equilibrium state, smooth solutions exist for all time and tends to the equilibrium constant state.

4. Binary mixtures and Maxwell Iteration

The multi–temperature model for mixtures, proposed within the context of extended thermodynamics, possesses desirable properties such as hyperbolicity, symmetric form of governing equations ensuring well-posedness of

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initial-value problems and global existence of smooth solutions. It is therefore quite natural to investigate its relation to the mixture model proposed in classical thermodynamics. There is a substantial difference between classical and extended approach. Diffusion flux and flux of internal energy are given by constitutive equations in classical thermodynamics, whereas in extended one they represent variables determined by the set of differential equations. Moreover, new model have the temperatures of constituents as additional independent variables whose evolution is governed by the balance laws for energy. To get a first impression about relation between classical and extended MT model a special case of binary mixture will be investigated.

In the case of binary non–reacting mixture of Euler fluids mathematical model could be written in the form[7]

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0;$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \operatorname{div}\left(\rho \mathbf{v} \otimes \mathbf{v} + p\mathbf{I} + \frac{1}{\rho c(1-c)}\mathbf{J} \otimes \mathbf{J}\right) = \mathbf{0};$$

$$\frac{\partial \left(\frac{1}{2}\rho v^2 + \rho \varepsilon\right)}{\partial t} + \operatorname{div}\left\{\left(\frac{1}{2}\rho v^2 + \rho \varepsilon + p\right)\mathbf{v} + \left(\frac{\mathbf{v} \cdot \mathbf{J}}{\rho c(1-c)} + \frac{1}{\alpha}\right)\mathbf{J}\right\} = 0;$$

$$\frac{\partial (\rho c)}{\partial t} + \operatorname{div}(\rho c\mathbf{v} + \mathbf{J}) = 0; \tag{20}$$

$$\frac{\partial (\rho c\mathbf{v} + \mathbf{J})}{\partial t} + \operatorname{div}\left\{\rho c\mathbf{v} \otimes \mathbf{v} + \frac{1}{\rho c}\mathbf{J} \otimes \mathbf{J} + \mathbf{v} \otimes \mathbf{J} + \mathbf{J} \otimes \mathbf{v} + \nu\mathbf{I}\right\} = \mathbf{m}_1;$$

$$\frac{\partial \left(\frac{1}{2}\rho c\left(\mathbf{v} + \frac{\mathbf{J}}{\rho c}\right)^2 + \rho ce\right)}{\partial t} +$$

$$\operatorname{div}\left\{\left(\frac{1}{2}\rho c\left(\mathbf{v} + \frac{\mathbf{J}}{\rho c}\right)^2 + \rho ce + \nu\right)\left(\mathbf{v} + \frac{\mathbf{J}}{\rho c}\right)\right\} = e_1,$$

where $c = \rho_1/\rho$, $\nu = p_1$ and $e = \varepsilon_1$. Diffusion flux vector is defined as

$$\mathbf{J} = \alpha \mathbf{q} = \rho_1 \mathbf{u}_1 = -\rho_2 \mathbf{u}_2,$$

where $\alpha = 1/(g_1 - g_2)$ is thermal inertia of the mixture with $g_\beta = \varepsilon_\beta + (p_\beta/\rho_\beta) + u_\beta^2/2$, $\beta = 1, 2$, being non-equilibrium enthalpies.

Thermodynamic process in binary mixture, governed by equations (20), is described by the field variables $(\rho, \mathbf{v}, T, c, \mathbf{J}, \Theta)$, with $\Theta = T_2 - T_1$. Therefore, $\mathbf{J}$ and Θ can be viewed as non-equilibrium variables in the sense of extended thermodynamics. Source terms $\mathbf{m}_1$ and e_1 are determined in ac-

cordance with general results (12)

$$\mathbf{m}_1 = \hat{\mathbf{m}}_1 = -\psi_{11} \left(\frac{1}{c\,T_1} - \frac{1}{(1-c)T_2} \right) \frac{\mathbf{J}}{\rho};$$

$$\hat{e}_1 = \theta_{11} \frac{\Theta}{T_1\,T_2}; \quad e_1 = \hat{\mathbf{m}}_1 \cdot \mathbf{v} + \hat{e}_1. \tag{21}$$

To reveal the relation between extended and classical model, a formal iterative scheme known as Maxwellian iteration will be applied.[3] The first iterates $\mathbf{J}^{(1)}$ and $\Theta^{(1)}$ are calculated from the right–hand sides of Eqs. $(20)_{5,6}$ by putting "zeroth" iterates — equilibrium values $\mathbf{J}^{(0)} = \mathbf{0}$ and $\Theta^{(0)} = 0$ on the left–hand sides. In the next step second iterates $\mathbf{J}^{(2)}$ and $\Theta^{(2)}$ are obtained from the right–hand sides of the same Eqs. by putting first $\mathbf{J}^{(1)}$ and $\Theta^{(1)}$ on their left–hand sides, an so on. Here, only the first iterates will be derived.

By applying the Maxwellian iteration scheme to Eq. $(20)_5$, after some simple manipulations with the use of Eqs. $(20)_{1,4}$ one obtains the first iterate of diffusion flux

$$\mathbf{J}^{(1)} = \tau_J \left(c\,\mathrm{grad}p - \mathrm{grad}\nu \right); \quad \tau_J = \frac{\rho c(1-c)}{\psi_{11}}, \tag{22}$$

where τ_J stands for the relaxation time of diffusion. In expanded form this relation reads

$$\mathbf{J}^{(1)} = k\,\tau_J \left\{ c(1-c) \left(\frac{1}{m_1} - \frac{1}{m_2} \right) (T\,\mathrm{grad}\rho + \rho\,\mathrm{grad}T) + \frac{\rho}{m(c)} T\mathrm{grad}c \right\}.$$

This equation can be recast into the form

$$\mathbf{J}^{(1)} = L_{11}\,\mathrm{grad} \left(\frac{\mu_2 - \mu_1}{T} \right) + L\,\mathrm{grad} \left(\frac{1}{T} \right),$$

well-known from TIP as Fick Law[3,15]. Phenomenological coefficients L_{11} and L are related to our field variables in the following way

$$L_{11} = \frac{\rho^2 c^2 (1-c)^2 T^2}{\psi_{11}}$$

$$L = -k \frac{\rho^2 c^2 (1-c)^2 T^3}{\psi_{11}} \left\{ \frac{\gamma_2}{\gamma_2 - 1} \frac{1}{m_2} - \frac{\gamma_1}{\gamma_1 - 1} \frac{1}{m_1} \right\}.$$

Treating Eq. $(20)_6$ in the same manner one obtains

$$\Theta^{(1)} = \tau_T \left(\frac{dT}{dt} + (\gamma_1 - 1)T\mathrm{div}\mathbf{v} \right); \quad \tau_T = \frac{k}{m_1(\gamma_1 - 1)} \frac{\rho c T^2}{\theta_{11}},$$

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where τ_T stands for relaxation time of non-equilibrium temperatures. Taking into account Eqs. $(20)_{3,4}$ and constitutive functions last equation can be reduced to

$$\Theta^{(1)} = L_\theta \left(\gamma_1 - \gamma_2 \right) \operatorname{div}\mathbf{v}. \tag{23}$$

where the corresponding phenomenological coefficient L_θ reads

$$L_\theta = \frac{k\rho c(1-c)T^3}{\theta_{11}(m_2(\gamma_2-1)c + m_1(\gamma_1-1)(1-c))}. \tag{24}$$

The equation (23), which seems to appear for the first time, expresses the temperature difference, here in the role of thermodynamic flux, as a linear function of divergence of velocity, i.e. thermodynamic force, as it is usual in non-equilibrium thermodynamics.

From (23) *it is very interesting to observe that in the classical TIP approach no possible difference of temperature between components arise if the ratio of the specific heats are the same.*

Equation (23) is a simplified version of general expression which can be obtained in the case of multi-component reacting mixtures. Being scalar quantities, the differences of temperatures $\Theta_b = T_n - T_b$ could also depend, in the first iteration, on so–called chemical "forces" which reflect the structure of chemical reactions taking place during the process.
Comparing our equations (1) with the ones obtained by the kinetic theory, we are able to evaluate the coefficients θ_{11} and ψ_{11}:

$$\psi_{11} = 2T \frac{m_1 m_2}{m_1 + m_2} \Gamma'_{12}; \qquad \theta_{11} = 3kT^2 \frac{m_1 m_2}{(m_1 + m_2)^2} \Gamma'_{12}$$

where Γ'_{12} represents the number of collision per density volume[16] .

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