

Mixture of Gases with Multi-temperature:

Identification of a macroscopic average temperature

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Abstract

In this paper we present the model of a mixture of gases in which each constituent has its own temperature. We review some recent results, in particular concerning the global existence of smooth solutions. Then we deduce a solution of the differential system under the assumption that the fields depend only on time. In this way, we are able to identify a possible natural *macroscopic average temperature*.

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1 The Mixture Theory

The description of simple homogeneous mixtures in the context of rational thermodynamics [1] relies on the postulate that each constituent obeys the same balance laws as a single fluid. They express rates of change of mass, momentum and energy with appropriate production terms due to mutual

interaction of the constituents:

$$\begin{aligned} \frac{\partial \rho_\alpha}{\partial t} + \operatorname{div}(\rho_\alpha \mathbf{v}_\alpha) &= \tau_\alpha \\ \frac{\partial(\rho_\alpha \mathbf{v}_\alpha)}{\partial t} + \operatorname{div}(\rho_\alpha \mathbf{v}_\alpha \otimes \mathbf{v}_\alpha - \mathbf{t}_\alpha) &= \mathbf{m}_\alpha, \quad (\alpha = 1, 2, \dots, n) \\ \frac{\partial \left(\frac{1}{2} \rho_\alpha v_\alpha^2 + \rho_\alpha \varepsilon_\alpha \right)}{\partial t} + \operatorname{div} \left\{ \left(\frac{1}{2} \rho_\alpha v_\alpha^2 + \rho_\alpha \varepsilon_\alpha \right) \mathbf{v}_\alpha - \mathbf{t}_\alpha \mathbf{v}_\alpha + \mathbf{q}_\alpha \right\} &= e_\alpha, \end{aligned} \quad (1)$$

being ρ_α the density, \mathbf{v}_α the velocity, ε_α the internal energy, \mathbf{t}_α the stress tensor and \mathbf{q}_α the heat flux of the α constituent of the mixture. The production terms must satisfy the following relations:

$$\sum_{\alpha=1}^n \tau_\alpha = 0; \quad \sum_{\alpha=1}^n \mathbf{m}_\alpha = \mathbf{0}; \quad \sum_{\alpha=1}^n e_\alpha = 0, \quad (2)$$

due to the global conservation of mass, momentum and energy of the mixture.

In fact, one may obtain from (1) the conservation laws of total mass, total momentum and total energy of the mixture through summation of the corresponding balance laws (1), after the introduction of following quantities:

$$\begin{aligned} \rho &= \sum_{\alpha=1}^n \rho_\alpha && \text{total mass density,} \\ \mathbf{v} &= \frac{1}{\rho} \sum_{\alpha=1}^n \rho_\alpha \mathbf{v}_\alpha && \text{mixture velocity,} \\ \varepsilon &= \varepsilon^I + \frac{1}{2\rho} \sum_{\alpha=1}^n \rho_\alpha u_\alpha^2 && \text{internal energy,} \\ \mathbf{t} &= \sum_{\alpha=1}^n (\mathbf{t}_\alpha - \rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha) && \text{stress tensor,} \\ \mathbf{q} &= \sum_{\alpha=1}^n \{ \mathbf{q}_\alpha + \rho_\alpha (\varepsilon_\alpha + \frac{1}{2} u_\alpha^2) \mathbf{u}_\alpha - \mathbf{t}_\alpha \mathbf{u}_\alpha \} && \text{flux of internal energy,} \\ \mathbf{u}_\alpha &= \mathbf{v}_\alpha - \mathbf{v} \quad \left(\sum_{\alpha=1}^n \rho_\alpha \mathbf{u}_\alpha = \mathbf{0} \right) && \text{diffusion velocity,} \\ \varepsilon^I &= \frac{1}{\rho} \sum_{\alpha=1}^n \rho_\alpha \varepsilon_\alpha && \begin{array}{l} \text{intrinsic} \\ \text{internal energy.} \end{array} \end{aligned} \quad (3)$$

The conservation laws read as the ones of a single fluid:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) &= 0; \\ \frac{\partial(\rho \mathbf{v})}{\partial t} + \operatorname{div}(\rho \mathbf{v} \otimes \mathbf{v} - \mathbf{t}) &= \mathbf{0}; \\ \frac{\partial \left(\frac{1}{2} \rho v^2 + \rho \varepsilon \right)}{\partial t} + \operatorname{div} \left\{ \left(\frac{1}{2} \rho v^2 + \rho \varepsilon \right) \mathbf{v} - \mathbf{t} \mathbf{v} + \mathbf{q} \right\} &= 0. \end{aligned} \quad (4)$$

In ordinary thermodynamics of mixtures, the state of the mixture is determined by $n + 4$ fields: the mass densities ρ_α , the mixture velocity \mathbf{v} and the common temperature T . Therefore, $n + 4$ field equations have to be chosen to describe thermodynamic processes. These are balance laws of mass $(1)_1$ and conservation laws of momentum and energy $(4)_{2,3}$. Usually, one of the balance laws $(1)_1$ is replaced by the conservation law $(4)_1$. Consequently, one may choose ρ , \mathbf{v} , T and $n - 1$ independent concentration variables c_α as field variables, where:

$$c_\alpha = \frac{\rho_\alpha}{\rho}; \quad \sum_{\alpha=1}^n c_\alpha = 1. \quad (5)$$

This model is characterized by an important shortcoming due to its constitutive assumptions (see [2], Chapter 5): it predicts infinite speed of propagation of disturbances in concentration also when viscosity and heat conductivity are neglected.

This problem has been successfully solved by the Rational Extended Thermodynamics [2] that describes thermodynamic processes by the $4n + 1$ fields of mass densities ρ_α , constituent velocities \mathbf{v}_α , and common single temperature T (ST model). The field equations are thus the balance laws of mass and momentum $(1)_{1,2}$ and the conservation law of energy $(4)_3$. Like in the previous case, conservation laws of mass and momentum $(4)_{1,2}$ may replace the corresponding balance laws for one constituent. Thus, one may choose ρ , \mathbf{v} , T , and $n - 1$ independent concentrations c_α and diffusion velocities \mathbf{u}_α as field variables. Finally, one may consider a model of a mixture in which each constituent has its own temperature. This assumption leads to a multi-temperature model of the mixture (MT model). In such a situation we have $5n$ variables ρ_α , \mathbf{v}_α and T_α and all the $5n$ balance laws (1) are needed for the description of the thermodynamic processes. Using the same idea as in previous cases, balance laws for one of the constituents, say n , could be replaced by conservation laws (4).

With this choice, the governing equations for the n -component mixture read:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) &= 0; \\
\frac{\partial(\rho \mathbf{v})}{\partial t} + \operatorname{div}(\rho \mathbf{v} \otimes \mathbf{v} - \mathbf{t}) &= \mathbf{0} \\
\frac{\partial \left(\frac{1}{2} \rho v^2 + \rho \varepsilon \right)}{\partial t} + \operatorname{div} \left\{ \left(\frac{1}{2} \rho v^2 + \rho \varepsilon \right) \mathbf{v} - \mathbf{t} \mathbf{v} + \mathbf{q} \right\} &= 0; \\
\frac{\partial \rho_b}{\partial t} + \operatorname{div}(\rho_b \mathbf{v}_b) &= \tau_b; \\
\frac{\partial(\rho_b \mathbf{v}_b)}{\partial t} + \operatorname{div}(\rho_b \mathbf{v}_b \otimes \mathbf{v}_b - \mathbf{t}_b) &= \mathbf{m}_b; \\
\frac{\partial \left(\frac{1}{2} \rho_b v_b^2 + \rho_b \varepsilon_b \right)}{\partial t} + \operatorname{div} \left\{ \left(\frac{1}{2} \rho_b v_b^2 + \rho_b \varepsilon_b \right) \mathbf{v}_b - \mathbf{t}_b \mathbf{v}_b + \mathbf{q}_b \right\} &= e_b \\
b &= 1, \dots, n-1.
\end{aligned} \tag{6}$$

In a recent paper [3] we have proved that requiring the Galilean invariance for the system (6) the production terms in (6) depend explicitly on velocity \mathbf{v} through:

$$\begin{aligned}
\tau_b &= \hat{\tau}_b; \\
m_b^j &= \hat{\tau}_b v^j + \hat{m}_b^j; \\
e_b &= \hat{\tau}_b \frac{v^2}{2} + \hat{m}_b^k v_k + \hat{e}_b.
\end{aligned} \tag{7}$$

where the hat indicates the corresponding internal counterparts independent of the velocity (objective quantities).

We consider now the simple case of a mixture of Eulerian perfect ideal fluids, i.e. neither viscous nor heat-conducting, i.e.:

$$\mathbf{t}_\alpha = -p_\alpha \mathbf{I}, \quad \mathbf{q}_\alpha = \mathbf{0} \quad ; \quad (\alpha = 1, \dots, n). \tag{8}$$

$$p_\alpha = \frac{k}{m_\alpha} \rho_\alpha T_\alpha; \quad \varepsilon_\alpha = \frac{p_\alpha}{\rho_\alpha (\gamma_\alpha - 1)}; \quad (\alpha = 1, \dots, n), \tag{9}$$

where k is the Boltzmann constant, m_α is the atomic mass of the α -constituent, p_α is the pressure of the α -constituent.

As a consequence of this assumption the stress tensor and the heat flux (3) are reduced to:

$$\begin{aligned}
\mathbf{t} &= -p \mathbf{I} - \sum_{\alpha=1}^n (\rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha); \\
\mathbf{q} &= \sum_{\alpha=1}^n \left\{ \rho_\alpha \left(\varepsilon_\alpha + \frac{1}{2} u_\alpha^2 \right) + p_\alpha \right\} \mathbf{u}_\alpha,
\end{aligned} \tag{10}$$

where

$$p = \sum_{\alpha=1}^n p_{\alpha} \quad (11)$$

is the total pressure.

The compatibility with the entropy principle permits to determine explicitly the internal productions [3]:

$$\begin{aligned} \hat{\tau}_b &= - \sum_{c=1}^{n-1} \varphi_{bc} \left(\frac{\mu_c - \frac{1}{2}u_c^2}{T_c} - \frac{\mu_n - \frac{1}{2}u_n^2}{T_n} \right); \\ \hat{\mathbf{m}}_b &= - \sum_{c=1}^{n-1} \psi_{bc} \left(\frac{\mathbf{u}_c}{T_c} - \frac{\mathbf{u}_n}{T_n} \right); \quad \hat{e}_b = - \sum_{c=1}^{n-1} \theta_{bc} \left(-\frac{1}{T_c} + \frac{1}{T_n} \right), \end{aligned} \quad (12)$$

where $\mu_{\alpha} = \varepsilon_{\alpha} - T_{\alpha}S_{\alpha} + p_{\alpha}/\rho_{\alpha}$, $\alpha = 1, \dots, n$ are chemical potentials of the constituents, while φ_{bc} , ψ_{bc} and θ_{bc} are positive definite matrix.

2 Qualitative Analysis

The MT model for mixtures is a particular case of a system of balance laws

$$\frac{\partial \mathbf{F}^0}{\partial t} + \frac{\partial \mathbf{F}^i}{\partial x} = \mathbf{F}; \quad (13)$$

where \mathbf{F}^0 , \mathbf{F}^i and \mathbf{F} are \mathbf{R}^N vectors. The system (13) is dissipative due to the presence of the productions that satisfy the entropy principle. On the other hand the system (6) is of mixed type since five equations are conservation laws while the rest of the system consisted of true balance laws

$$\mathbf{F}(\mathbf{u}) \equiv \begin{pmatrix} 0 \\ \mathbf{g}(\mathbf{u}) \end{pmatrix}.$$

For this class of systems the coupling condition of Shizuta and Kawashima (K-condition) [4] can ensure the global existence of smooth solutions, meaning that dissipation in balance laws prevails the hyperbolicity of conservation laws. Actually, the K-condition reads: *In the equilibrium manifold any characteristic eigenvector is not in the null space of $\nabla \mathbf{F}$* , i.e.

$$\nabla \mathbf{F} \cdot \mathbf{d}^{(i)} \Big|_E \neq 0 \quad \forall i = 1, \dots, N. \quad (14)$$

where $\mathbf{d}^{(i)}$ are the right-eigenvectors of the hyperbolic system (13):

$$(\mathbf{A}_n - \lambda \mathbf{I}) \mathbf{d} = 0, \quad (15)$$

$\mathbf{A}_n = \mathbf{A}^i n_i$, $\mathbf{A}^i = \nabla \mathbf{F}^i$, $\mathbf{u} = \mathbf{F}^0$, $\nabla = \partial/\partial \mathbf{u}$, $\mathbf{n} \equiv (n_i)$ is a generic unit vector and E stands for the equilibrium state.

It has been proven recently that if the system of balance laws (13) is endowed with a convex entropy law, and it is dissipative, then the K-condition becomes a sufficient condition for the existence of global smooth solutions provided that the initial data are sufficiently smooth [5, 6]. Furthermore, Ruggeri and Serre [7] proved the stability of constant states in one-dimensional case.

After an extensive calculation [3] it has been shown that K-condition is satisfied for the MT model (6) of mixtures together with the entropy convexity and that, according to above mentioned results, the following statement is valid:

If the initial data of the MT model are perturbations of equilibrium state, smooth solutions exist for all time and tends to the equilibrium constant state.

However, this nice behavior is not valid for the ST model [8, 9, 10] and this is another important indication that the MT model is more realistic with respect to the ST one.

Moreover in [3] we have show that ST model, within the framework of hyperbolic systems, represents a *principal subsystem* of the MT model in the sense of a definition due to Boillat and Ruggeri [11].

3 Binary mixture

Let us consider the simple case of a binary mixture without chemical reaction $\tau_\alpha = 0$. To understand the difference between the ST model and the MT model we consider solutions of (1) in which the fields depend only on time. With these assumptions balance laws (1) read:

$$\frac{d\rho_1}{dt} = 0; \quad \frac{d\rho_2}{dt} = 0; \quad (16)$$

$$\rho_1 \frac{d\mathbf{v}_1}{dt} = \mathbf{m}_1; \quad \rho_2 \frac{d\mathbf{v}_2}{dt} = -\mathbf{m}_1; \quad (17)$$

$$\frac{d}{dt} \left(\frac{1}{2} \rho_1 v_1^2 + \rho_1 \varepsilon_1 \right) = e_1; \quad \frac{d}{dt} \left(\frac{1}{2} \rho_2 v_2^2 + \rho_2 \varepsilon_2 \right) = -e_1. \quad (18)$$

From (16) and (17) we have:

$$\rho_1 = \text{const.}; \quad \rho_2 = \text{const.}; \quad \rho \mathbf{v} = \text{const.}$$

and the constant velocity $\mathbf{v} = 0$ can be chosen without loss of generality due to Galilean invariance.

From (18) and (9) we obtain:

$$\frac{1}{2}\rho_1 v_1^2 + \frac{1}{2}\rho_2 v_2^2 + \frac{k}{m_1} \frac{\rho_1 T_1}{\gamma_1 - 1} + \frac{k}{m_2} \frac{\rho_2 T_2}{\gamma_2 - 1} = E = \text{const.} \quad (19)$$

Taking into account that in equilibrium $\mathbf{v}_1 = \mathbf{v}_2 = 0$, $T_1 = T_2 = T_0$, the following value of the constant E in (19) is obtained:

$$E = k \left(\frac{1}{m_1} \frac{\rho_1}{\gamma_1 - 1} + \frac{1}{m_2} \frac{\rho_2}{\gamma_2 - 1} \right) T_0. \quad (20)$$

From (19) and (20) follows that there exists an *average diffusive temperature* T which remains constant during the process:

$$T = T_0 = \frac{\frac{1}{2}\rho_1 v_1^2 + \frac{1}{2}\rho_2 v_2^2 + \frac{k}{m_1} \frac{\rho_1}{\gamma_1 - 1} T_1 + \frac{k}{m_2} \frac{\rho_2}{\gamma_2 - 1} T_2}{\frac{k}{m_1} \frac{\rho_1}{\gamma_1 - 1} + \frac{k}{m_2} \frac{\rho_2}{\gamma_2 - 1}}. \quad (21)$$

Furthermore, if processes not far from the equilibrium state are to be considered, diffusive quadratic terms in (21) could be neglected and the following average temperature is obtained:

$$T = T_0 = \frac{\rho_1 m_2 (\gamma_2 - 1) T_1 + \rho_2 m_1 (\gamma_1 - 1) T_2}{\rho_1 m_2 (\gamma_2 - 1) + \rho_2 m_1 (\gamma_1 - 1)}. \quad (22)$$

Therefore (22), at least for this kind of processes, permits to identify a possible natural *macroscopic average temperature* that coincides with the one of ST model and remains constant for all time.

Linearizing equations (16-18) in the neighborhood of an equilibrium state and taking into account (12), one obtains:

$$\frac{d\mathbf{v}_1}{dt} = -\frac{1}{\tau_v} \mathbf{v}_1, \quad \frac{d\mathbf{v}_2}{dt} = -\frac{1}{\tau_v} \mathbf{v}_2, \quad (23)$$

$$\frac{dT_1}{dt} = -\frac{1}{\tau_T} (T_1 - T_0); \quad \frac{dT_2}{dt} = -\frac{1}{\tau_T} (T_2 - T_0)$$

with

$$\tau_T = \frac{k\rho_1\rho_2 T_0^2}{\theta_{11}^0 (\rho_1 m_2 (\gamma_2 - 1) + \rho_2 m_1 (\gamma_1 - 1))}, \quad \tau_v = \frac{\rho_1 \rho_2 T_0}{\rho \psi_{11}^0} \quad (24)$$

(θ_{11}^0 and ψ_{11}^0 indicate the constant equilibrium values of θ_{11} and ψ_{11} respectively). Initial data $\mathbf{v}_1(0)$, $\mathbf{v}_2(0)$ are taken in accordance with constraint $\mathbf{v} = 0$, i.e.

$$\rho_1 \mathbf{v}_1(0) + \rho_2 \mathbf{v}_2(0) = 0 \quad (25)$$

while (22) provides the value of equilibrium temperature T_0 in terms of $T_1(0)$ and $T_2(0)$:

$$T_0 = \frac{\rho_1 m_2 (\gamma_2 - 1) T_1(0) + \rho_2 m_1 (\gamma_1 - 1) T_2(0)}{\rho_1 m_2 (\gamma_2 - 1) + \rho_2 m_1 (\gamma_1 - 1)}. \quad (26)$$

Consequently the solution of (23) reads:

$$\mathbf{v}_1 = \mathbf{v}_1(0) e^{-\frac{t}{\tau_v}}; \quad \mathbf{v}_2 = \mathbf{v}_2(0) e^{-\frac{t}{\tau_v}}; \quad (27)$$

$$T_1 = T_0 + (T_1(0) - T_0) e^{-\frac{t}{\tau_T}}; \quad T_2 = T_0 + (T_2(0) - T_0) e^{-\frac{t}{\tau_T}}; \quad (28)$$

with (25) and (26). One may conclude that for t large, \mathbf{v}_1 and \mathbf{v}_2 tend to zero with relaxation time τ_v , while T_1 and T_2 tend to the equilibrium temperature T_0 with relaxation time τ_T .

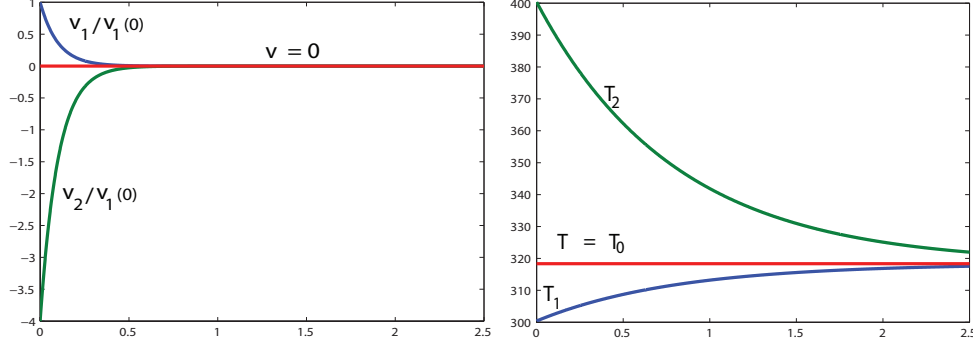


Fig. 1: The behavior of normalized velocities and temperatures of the constituents versus time ($m_2/m_1 = 1.14$, $T_1(0) = 300$ K, $T_2(0) = 400$ K, $c = \rho_1/\rho = 0.8$, $\gamma_1 = \gamma_2 = 7/5$, $\tau_v = 0.1$).

To compare the relaxation times τ_v and τ_T the following relations from the kinetic theory may be recalled [12]:

$$\theta_{11} = \frac{3m_1m_2}{(m_1 + m_2)} kT_0^2 \Gamma'_{12}, \quad \psi_{11} = 2 \frac{m_1m_2}{m_1 + m_2} \Gamma'_{12} \quad (29)$$

where Γ'_{12} is the number of collisions per unit of volume. From (24) and (29) one obtains:

$$\frac{\tau_T}{\tau_v} = \frac{2}{3} \frac{(m_1 + m_2)\rho}{m_1(\gamma_1 - 1)\rho_2 + m_2(\gamma_2 - 1)\rho_1}. \quad (30)$$

For the mixture of monatomic gases, $\gamma_1 = \gamma_2 = 5/3$, we have:

$$\frac{\tau_T}{\tau_v} = \frac{\rho(m_1 + m_2)}{\rho_1 m_2 + \rho_2 m_1} \geq 1 \quad (31)$$

leading to $\tau_T \geq \tau_v$. This inequality indicates that diffusion processes are attenuated faster than the temperature difference, providing another proof of superiority of the MT model to the ST one. The Fig.1 represents the sketch of the solutions (27) and (28) for data corresponding to the air constituted by 80% of Nitrogen and by 20% of Oxygen.

The necessity for a macroscopic temperature comes from the fact that it is practically impossible to measure the temperatures of the constituents. That was probably the main reason for the domination of the ST model in literature. Nevertheless, the ST model—as we have seen—cannot be regarded as realistic one, in particular when the atomic masses of each constituent are different. Therefore we can use the more realistic MT model and choose T given by (22) as possible *average macroscopic temperature*.

Observe that in the case of a generic n -component mixture (22) is generalized to:

$$T = \frac{\sum_{\alpha=1}^n \frac{\rho_{\alpha}}{m_{\alpha}(\gamma_{\alpha} - 1)} T_{\alpha}}{\sum_{\beta=1}^n \frac{\rho_{\beta}}{m_{\beta}(\gamma_{\beta} - 1)}}. \quad (32)$$

As in the case of a binary mixture, (32) is obtained by requirement that the global intrinsic internal energy of MT theory is equal to the one of the ST theory:

$$\varepsilon_{MT}^I = \varepsilon_{ST}^I. \quad (33)$$

In the class of solutions in which the fields depend only on time, the choice of (32) seems to be natural. The macroscopic average temperature so obtained remains constant for processes not far from the equilibrium, being in agreement with the ST theory and the physical expectation as well. Of course, for generic processes depending on space and time there are in principle several other possibilities to define an average temperature and the choice (32) has to be compared with them and to be tested if in agreement with experiments.

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