FORCED TRANSVERSE VIBRATIONS OF ELASTIC BEAMS AND THEIR DYNAMIC ABSORPTION

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1. Introduction

It is well known that forced vibrations of one-degree-of-freedom dynamical systems cannot be damped by means of ordinary viscous damper. Therefore, a very efficient method of dynamic absorption was developed in order to reduce the amplitude of forced vibrations as much as possible. Firstly, it was developed as a technical device, and later on put on firm theoretical ground. For a good historical account, as well as review of different applications, see Hunt [1].

The simplest model of dynamic absorber consists of a coupled lumped parameter system with two degrees of freedom, where the primary element is exposed to time-periodic forcing, while secondary element is dynamic absorber. Corresponding mathematical model reads:

$$\begin{aligned} &\ddot{x}_1 + \lambda_1 x_1 - \kappa(x_2 - x_1) = f_1(t), \\ &\mu\ddot{x}_2 + \mu\lambda_2 x_2 + \kappa(x_2 - x_1) = 0, \end{aligned} \tag{1}$$

where $\lambda_i = c_i/m_i$, $i = 1,2$, $\mu = m_2/m_1$, $\kappa = k/m_1$ and $f_1(t) = F_1(t)/m_1 = f_0 \sin(\Omega t)$. The amplitude of pure forced vibrations of primary element will be completely suppressed, e.g. zero, if the following condition of dynamic absorption is fulfilled:

$$\kappa + \mu(\lambda_2 - \Omega^2) = 0. \tag{2}$$

Thus, the parameters of the secondary element and interconnecting spring can be adjusted in such a way that amplitude of the primary one becomes negligible. Although this result seems to be very optimistic, in reality primary element does not remain in the state of rest. First, if the system was initially in the state of rest there will exist another part of the solution, other than pure forced oscillations, so that amplitude of motion of primary element is not negligible anymore, but only reduced to some reasonable amount. Second, model (1) ignores the presence of dissipative forces, usual in almost all systems, which prevents complete suppression of the amplitude of pure forced vibrations - see for example Den Hartog [2]. Third, it is a rather difficult task to reduce a real physical system to the model with lumped parameters such as (1). Usually, one deals either with many degrees of freedom systems, or with nonlinear models.

After the pioneering work of Ormondroyd and Den Hartog [3] an almost endless stream of scientific and technical literature appeared devoted to dynamic vibration absorbers and other similar problems. Some of the recent studies, like Hsueh [4], discuss multi-degree-of-freedom systems with multiple dynamic absorbers. Another line of investigation develops new types of active dynamic absorbers - see Burdisso and Heilmann [5] and Shi *et al.* [6]. Finaly, an important question of mathematical methods for free and forced vibrations of continuous systems is still a matter of scientific interest, like in Lueschen and Bergman [7] and Sakiyama *et al.* [8].

In the present work we shall analyze the problem of dynamic absorption of forced transverse vibrations of a simply supported elastic beam. It is inspired mainly by the work of Oniszczuk [9], [10], [11] who discussed transverse vibrations of elastically connected double-string and double-beam systems. Our intention is to put forward the analysis of forced vibrations of a double-beam system and derive the formal solution of initial-boundary value problem in such a way that direct correspondence with two-degrees-of-freedom system can be established. An influence of the secondary element to the eigenfrequencies of the primary one will be discussed, and we shall perform a study of dynamic absorber for various distributions of forcing term and present a criterion of its efficiency.

2. The model

Let us analyze two simply supported beams of the same length l interconnected by an elastic element of Winkler type. In the literature they are called sandwiched beams, layered or composite structures. We shall suppose that materials of the beams are Hookean and that material and geometrical properties (elasticity modulus E, material density ρ, cross-sectional area A, principal moment of inertia I) of the elements are different, but constant along their axes. Let us suppose also that the axes of beams are straight in the undeformed state and that vibrations occur only in the vertical plane without damping. If we assume that deflections of beams are small the following linear model can be obtained using d'Alembert's principle or variational methods:

$$\begin{aligned} A_1\rho_1 \frac{\partial^2 y_1}{\partial t^2} + E_1 I_1 \frac{\partial^4 y_1}{\partial x^4} - k(y_2 - y_1) &= F_1(x,t), \\ A_2\rho_2 \frac{\partial^2 y_2}{\partial t^2} + E_2 I_2 \frac{\partial^4 y_2}{\partial x^4} + k(y_2 - y_1) &= 0. \end{aligned} \tag{3}$$

where $x \in (0,l)$ and $t \in (0,\infty)$ are independent space and time variables, $y_i(x,t)$, $i = 1,2$, denote deflections of beams, k is constant stiffness of Winkler elastic element and $F_1(x,t)$ distributed load which acts on primary element. Governing equations (3) are adjoined with boundary conditions:

$$y_i(0,t) = y_i(l,t) = 0; \quad \frac{\partial^2 y_i}{\partial x^2}(0,t) = \frac{\partial^2 y_i}{\partial x^2}(l,t) = 0; \quad i = 1,2, \tag{4}$$

while initial state of the system is governed by initial conditions which will be introduced later.

In the following text we shall transform our model (3)-(4) into dimensionless form by introduction of dimensionless space and time variables ξ and τ, and dimensionless deflection of beams $u_i(\xi,\tau)$, $i = 1,2$:

$$\xi = \frac{x}{l}; \quad \tau = t\sqrt{\frac{E_1 I_1}{A_1\rho_1 l^4}}; \quad u_i(\xi,\tau) = \frac{y_i(x,t)}{l}. \tag{5}$$

Now, dimensionless mathematical model of forced transverse vibrations of beams reads:

$$\begin{aligned} \frac{\partial^2 u_1}{\partial \tau^2} + \frac{\partial^4 u_1}{\partial \xi^4} - \kappa(u_2 - u_1) &= f_1(\xi,\tau), \\ \mu\frac{\partial^2 u_2}{\partial \tau^2} + EI\frac{\partial^4 u_2}{\partial \xi^4} + \kappa(u_2 - u_1) &= 0, \end{aligned} \tag{6}$$

where $\xi \in (0,1)$, $\tau \in (0,\infty)$, $f_1(\xi,\tau) = F_1(x,t)/E_1 I_1$ is dimensionless distributed load, $\mu = A_2\rho_2/A_1\rho_1$ is ratio of linear densities, $EI = E_2 I_2/E_1 I_1$ is ratio of flexural rigidities and $\kappa = kl^3/E_1 I_1$ is dimensionless stiffness of Winkler element. Corresponding boundary conditions (4) are transformed to:

$$u_i(0,\tau) = u_i(1,\tau) = 0; \quad \frac{\partial^2 u_i}{\partial \xi^2}(0,\tau) = \frac{\partial^2 u_i}{\partial \xi^2}(1,\tau) = 0; \quad i = 1,2, \tag{7}$$

while initial conditions have the following general form:

$$u_i(\xi,0) = u_{i0}(\xi); \quad \frac{\partial u_i}{\partial \tau}(\xi,0) = v_{i0}(\xi); \quad i = 1,2. \tag{8}$$

In this model beam *1* is primary element and beam *2* plays the role of continuous dynamic absorber. In the sequel our analysis will be based upon formal solution of initial-boundary-value problem (6)-(8) using the method of eigenfunction expansion.

3. Formal solution of governing equations

Let us transform the system of governing equations (6) by separating the principal part of the system (left-hand side) from the non-homogeneous part (right-hand side):

$$\frac{\partial^2 u_1}{\partial \tau^2} + \frac{\partial^4 u_1}{\partial \xi^4} + \kappa u_1 = \kappa u_2 + f_1(\xi, \tau),$$
$$\mu \frac{\partial^2 u_2}{\partial \tau^2} + EI \frac{\partial^4 u_2}{\partial \xi^4} + \kappa u_2 = \kappa u_1. \tag{9}$$

We shall solve the associated eigenvalue problem arising from the principal part of equations (9):

$$\frac{\partial^2 u_1}{\partial \tau^2} + \frac{\partial^4 u_1}{\partial \xi^4} + \kappa u_1 = 0; \quad \mu \frac{\partial^2 u_2}{\partial \tau^2} + EI \frac{\partial^4 u_2}{\partial \xi^4} + \kappa u_2 = 0, \tag{10}$$

together with boundary conditions (7). System (10) describes two uncoupled beams on an elastic foundation of Winkler type. Now we can apply the Fourier's method of separation of variables $u_i(\xi, \tau) = w_i(\xi) T_i(\tau)$ and obtain the corresponding eigenvalue problems:

$$\frac{d^4 w_1}{d\xi^4} - \lambda_1 w_1 = 0; \quad EI \frac{d^4 w_2}{d\xi^4} - \mu \lambda_2 w_2 = 0;$$
$$w_i(0) = w_i(1) = 0; \quad \frac{d^2 w_i}{d\xi^2}(0) = \frac{d^2 w_i}{d\xi^2}(1) = 0; \quad i = 1,2. \tag{11}$$

It is a simple matter to deduce from (11) the following set of eigenvalue-eigenfunction pairs:

$$\lambda_{1n} = n^4 \pi^4, \quad w_{1n}(\xi) = \sqrt{2} \sin\left(\lambda_{1n}{}^{1/4} \xi\right);$$
$$\lambda_{2n} = n^4 \pi^4 \frac{EI}{\mu}, \quad w_{2n}(\xi) = \sqrt{2} \sin\left(\left(\lambda_{2n} \frac{\mu}{EI}\right)^{1/4} \xi\right); \tag{12}$$

for $n = 1,2,\ldots$. Note that normalized eigenfunctions of both eigenvalue problems have the same form: $w_{1n}(\xi) = w_{2n}(\xi) = \sqrt{2} \sin(n\pi\xi)$. Therefore, the eigenfunction expansion of formal solutions read:

$$u_i(\xi, \tau) = \sum_{n=1}^{\infty} T_{in}(\tau) w_{in}(\xi) = \sum_{n=1}^{\infty} \sqrt{2} T_{in}(\tau) \sin(n\pi\xi), \tag{13}$$

where $T_{in}(\tau)$ are the functions describing their time evolution. By substituting (13) in (9) we obtain the family of coupled second-order ordinary differential equations for determination of functions $T_{in}(\tau)$:

$$\ddot{T}_{1n} + \lambda_{1n} T_{1n} - \kappa (T_{2n} - T_{1n}) = f_{1n}(\tau);$$
$$\mu \ddot{T}_{1n} + \mu \lambda_{2n} T_{1n} + \kappa (T_{2n} - T_{1n}) = 0, \tag{14}$$

where an overdot denotes differentiation with respect to τ and $f_{1n}(\tau)$ are the coefficients in eigenfunction expansion of forcing term $f_1(\xi, \tau) = \sum_{n=1}^{\infty} \sqrt{2} f_{1n}(\tau) \sin(n\pi\xi)$.

Here we arrive to our first important result. Although the application of eigenfunction expansion is rather common place in analyses of linear models od dynamic absorbers, there have not been shown such a clear formal correspondence between two-degree-of-freedom system and continuous absorbers like in equations (1) and (14). Therefore, we are in a position to derive some important results for the system with time-periodic forcing $f_{1n}(\tau)$ by pure analogy, but we shall rather postpone them until the complete formal solution is obtained. It will be just pointed out that each member of the family (14), i.e. system obtained for a particular value of *n*, determines the time evolution of the corresponding mode of vibration described by a particular eigenfunction.

In order to derive formal solution of governing equations, which will serve as a basis for the study of continuous dynamic absorber, we shall apply the method of variation of constants to the family of systems (14) and obtain the following result:

$$u_1(\xi, \tau) = \sum_{n=1}^{\infty} \sqrt{2} \left(A_{1n} \sin(\omega_{n1}\tau) + B_{1n} \cos(\omega_{n1}\tau) + C_{1n} \sin(\omega_{n2}\tau) + D_{1n} \cos(\omega_{n2}\tau)\right) \sin(n\pi\xi);$$
$$u_2(\xi, \tau) = \sum_{n=1}^{\infty} \sqrt{2} \left(\alpha_{1n} A_{1n} \sin(\omega_{n1}\tau) + \alpha_{1n} B_{1n} \cos(\omega_{n1}\tau) + \alpha_{2n} C_{1n} \sin(\omega_{n2}\tau) + \alpha_{2n} D_{1n} \cos(\omega_{n2}\tau)\right) \sin(n\pi\xi), \tag{15}$$

where ω_{n1} and ω_{n2} are eigenfrequences obtained as solutions of biquadratic frequency equation:

$$\mu\omega^4 - \left(\kappa + \mu\kappa + \mu\lambda_{1n} + \mu\lambda_{2n}\right)\omega^2 + \kappa\lambda_{1n} + \mu\kappa\lambda_{2n} + \mu\lambda_{1n}\lambda_{2n} = 0, \tag{16}$$

and α_{1n} and α_{2n} are coefficients of form:

$$\alpha_{1n} = \frac{\kappa + \lambda_{1n} - \omega_{n1}^2}{\kappa} = \frac{\kappa}{\kappa + \mu\lambda_{2n} - \mu\omega_{n1}^2}; \quad \alpha_{2n} = \frac{\kappa + \lambda_{1n} - \omega_{n2}^2}{\kappa} = \frac{\kappa}{\kappa + \mu\lambda_{2n} - \mu\omega_{n2}^2}. \tag{17}$$

Time-dependent coefficients A_{1n}, B_{1n}, C_{1n} and D_{1n} are determined in the course of variation of constants and they contain terms which guarantee satisfying both initial conditions (8) and right-hand sides of the systems (14), and consequently of the system (6):

$$\begin{aligned}
A_{1n} &= -\frac{1}{\omega_{n1}\left(\alpha_{1n} - \alpha_{2n}\right)}\left(\alpha_{2n}\int_0^1 \sqrt{2}v_{10}(\xi)\sin(n\pi\xi)d\xi - \int_0^1 \sqrt{2}v_{20}(\xi)\sin(n\pi\xi)d\xi\right) \\
&\quad - \frac{\kappa + \mu\left(\lambda_{2n} - \omega_{n1}^2\right)}{\mu\omega_{n1}\left(\omega_{n1}^2 - \omega_{n2}^2\right)}\int_0^\tau f_{1n}(s)\cos(\omega_{n1}s)ds; \\
B_{1n} &= -\frac{1}{\alpha_{1n} - \alpha_{2n}}\left(\alpha_{2n}\int_0^1 \sqrt{2}u_{10}(\xi)\sin(n\pi\xi)d\xi - \int_0^1 \sqrt{2}u_{20}(\xi)\sin(n\pi\xi)d\xi\right) \\
&\quad + \frac{\kappa + \mu\left(\lambda_{2n} - \omega_{n1}^2\right)}{\mu\omega_{n1}\left(\omega_{n1}^2 - \omega_{n2}^2\right)}\int_0^\tau f_{1n}(s)\sin(\omega_{n1}s)ds; \\
C_{1n} &= \frac{1}{\omega_{n2}\left(\alpha_{1n} - \alpha_{2n}\right)}\left(\alpha_{1n}\int_0^1 \sqrt{2}v_{10}(\xi)\sin(n\pi\xi)d\xi - \int_0^1 \sqrt{2}v_{20}(\xi)\sin(n\pi\xi)d\xi\right) \\
&\quad + \frac{\kappa + \mu\left(\lambda_{2n} - \omega_{n2}^2\right)}{\mu\omega_{n2}\left(\omega_{n1}^2 - \omega_{n2}^2\right)}\int_0^\tau f_{1n}(s)\cos(\omega_{n2}s)ds; \\
D_{1n} &= \frac{1}{\alpha_{1n} - \alpha_{2n}}\left(\alpha_{1n}\int_0^1 \sqrt{2}u_{10}(\xi)\sin(n\pi\xi)d\xi - \int_0^1 \sqrt{2}u_{20}(\xi)\sin(n\pi\xi)d\xi\right) \\
&\quad - \frac{\kappa + \mu\left(\lambda_{2n} - \omega_{n2}^2\right)}{\mu\omega_{n2}\left(\omega_{n1}^2 - \omega_{n2}^2\right)}\int_0^\tau f_{1n}(s)\sin(\omega_{n2}s)ds.
\end{aligned} \tag{18}$$

This completes the construction of formal solution of the initial-boundary value problem (6)-(8).

4. The dynamic absorber

In the analysis of dynamic absorber it will be assumed that the initial conditions (8) are homogeneous $u_{i0}(\xi) = 0$, $v_{i0}(\xi) = 0$ and that the forcing term describes harmonic excitation $f_1(\xi,\tau) = F(\xi)\sin(\Omega\tau)$. This reduces the general solution $(15)_1$ of the primary element to the form:
$\left(\Omega^2 - \omega_{n1}^2\right)$

$$\begin{aligned}
u_1(\xi,\tau) = \sum_{n=1}^{\infty} 2\left(\int_0^1 F(\xi)\sin(n\pi\xi)d\xi\right)\Bigg\{ & \underline{\underline{\frac{\kappa + \mu\left(\lambda_{2n} - \Omega^2\right)}{\mu\left(\Omega^2 - \omega_{n1}^2\right)\left(\Omega^2 - \omega_{n2}^2\right)}\sin(\Omega\tau)}} - \frac{\kappa + \mu\left(\lambda_{2n} - \omega_{n1}^2\right)}{\mu\left(\Omega^2 - \omega_{n1}^2\right)\left(\omega_{n1}^2 - \omega_{n2}^2\right)}\frac{\Omega}{\omega_{n1}}\sin(\omega_{n1}\tau) \\
& + \frac{\kappa + \mu\left(\lambda_{2n} - \omega_{n2}^2\right)}{\mu\left(\Omega^2 - \omega_{n2}^2\right)\left(\omega_{n1}^2 - \omega_{n2}^2\right)}\frac{\Omega}{\omega_{n2}}\sin(\omega_{n2}\tau)\Bigg\}\sin(n\pi\xi).
\end{aligned} \tag{19}$$

Underlined term describes pure forced vibrations of the beam and usually other time-dependent terms are dropped in the study of dynamic absorption. At first, we shall be focused on this expression in order to reveal complete analogy with two-degrees-of-freedom system.

First, since the solution is expressed in terms of infinite series the amplitude of forced vibrations cannot be completely suppressed. It could take negligible value *only in one mode*, while other amplitudes remain non-zero. Thus condition of dynamic absorption for the n^{th} mode of vibration reads:

$$\kappa + \mu\left(\lambda_{2n} - \Omega^2\right) = 0, \tag{20}$$

and it is formally equivalent to the condition (2). Second, it can be seen that distribution of the load $F(\xi)$ does not affect condition (20) at all, like in the case of lumped parameter system (1) where the amplitude of forcing term did not play any role in condition (2). It is excitation frequency Ω which is mandatory for the choice of parameters of the system that will guarantee dynamic absorption. Third, there is no mathematical criterion for the choice of the mode that has to be suppressed. This can be done only using physical arguments. It is usual to choose the first mode to be absorbed since it has the largest amplitude and carries the largest amount of total mechanical energy. In fact, the main idea of dynamic absorbers is not to absorb the mechanical energy by means of dissipation, but to push the system away from the resonant regime by attaching the secondary element. The outcome of this action is that the new eigenfrequences differ from the original excitation frequency, usually lying in the neighborhood of the lowest eigenfrequency of unabsorbed system.

To get deeper insight of the general analysis we shall present the results of numerical study for various types of the load. Namely, dynamic absorption will be analyzed for the following distributions of exetrnal load:

$$\begin{aligned}
f_1^{(1)}(\xi,\tau) &= \delta(\xi - 0.4)\sin(\Omega\tau);\\
f_1^{(2)}(\xi,\tau) &= 0.8\delta(\xi - 0.3)\sin(\Omega\tau) + 0.5\delta(\xi - 0.5)\sin(\Omega\tau - \beta);\\
f_1^{(3)}(\xi,\tau) &= \left(H(\xi - 0.3) - H(\xi - 0.5)\right)\sin(\Omega\tau);\\
f_1^{(4)}(\xi,\tau) &= 0.8\left(H(\xi - 0.25) - H(\xi - 0.35)\right)\sin(\Omega\tau)\\
&\quad + 0.5\left(H(\xi - 0.45) - H(\xi - 0.55)\right)\sin(\Omega\tau - \beta),
\end{aligned} \tag{21}$$

where $\delta(\xi)$ is Dirac-delta and $H(\xi)$ is Heaviside function modelling concentrated and uniformly distributed load, respectively. Dynamical parameter which is to be monitored is the amplitude of the L^2-norm of the complete solution (19), i.e. $\max\left\|u_1(\cdot,\tau)\right\|_{L^2(0,1)}$. In order to present results in a compact form we introduced dimensionless group $\Pi^2 = EI/\mu$ so that condition of dynamic absorber (20) is reduced to:

$$\kappa + \mu\left(n^4\pi^4\Pi^2 - \Omega^2\right) = 0. \tag{22}$$

We shall choose the value of Π to absorb the first mode of vibrations for the following values of system parameters $\mu = 1.0$, $\kappa = 10.0$, $\beta = \pi/3$, and excitation frequency $\Omega = 0.99\omega_{s1}$, where $\omega_{s1} = \pi^2$ represents the lowest dimensionless eigenfrequency of the single simply supported beam without Winkler element.

In Table 1 and Table 2 we have given a comparison of amplitudes of first six modes of a single beam and the absorbed one with secondary element and Winkler elastic layer. It can be seen that in all four cases the amplitude of the first mode is reduced to 10% of its original value, representing the remainder of the complete solution obtained after absorption of pure forced vibrations. At the same time, amplitudes of the other modes retain the same order of magnitude. Large suppression of the fifth mode in cases $f_1^{(1)}(\xi,\tau)$ and $f_1^{(3)}(\xi,\tau)$ can be explained by the presence of the node of the eigenfunction in the neighborhood of the load.

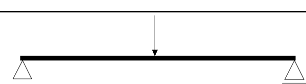
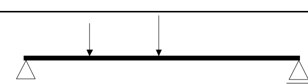
Type of the load				
	amplitude		amplitude	
modes	single beam	absorbed	single beam	absorbed
1	1.38069	0.112599	1.23781	0.118557
2	7.08756×10^{-4}	7.03523×10^{-4}	9.17433×10^{-4}	9.1066×10^{-4}
3	1.18255×10^{-4}	1.18216×10^{-4}	7.46482×10^{-5}	1.56925×10^{-4}
4	5.74934×10^{-5}	5.747×10^{-5}	2.84263×10^{-5}	2.84147×10^{-4}
5	5.9239×10^{-21}	5.92293×10^{-21}	1.93823×10^{-5}	2.63688×10^{-5}
6	1.09524×10^{-5}	1.09544×10^{-5}	5.41516×10^{-6}	5.41617×10^{-6}

Table 1: Cases $f_1^{(1)}(\xi,\tau)$ and $f_1^{(2)}(\xi,\tau)$

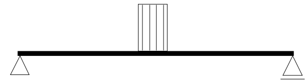
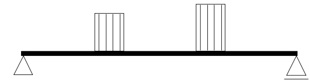
Type of the load				
	amplitude		amplitude	
modes	single beam	absorbed	single beam	absorbed
1	0.271617	0.0221511	0.144175	0.011807
2	1.32607×10^{-4}	1.31628×10^{-4}	9.02416×10^{-5}	8.95752×10^{-5}
3	2.03019×10^{-5}	2.02936×10^{-5}	1.71375×10^{-5}	1.51181×10^{-5}
4	8.70251×10^{-6}	8.69896×10^{-6}	2.65925×10^{-6}	2.65817×10^{-6}
5	6.83804×10^{-22}	6.83692×10^{-22}	2.37391×10^{-6}	2.37402×10^{-6}
6	1.10521×10^{-6}	1.10542×10^{-6}	4.64834×10^{-6}	4.6492×10^{-7}

Table 2: Cases $f_1^{(3)}(\xi,\tau)$ *and* $f_1^{(4)}(\xi,\tau)$

5. Conclusions

In this work we analyzed undamped continuous dynamic absorber consisted of two simply supported beams interconnected by the elastic layer of Winkler type. Using the specific procedure for construction of formal solution we established a formal correspondence of our problem and the classic lumped parameter absorber with two degrees of freedom. We showed that only one mode of pure forced vibrations of primary element can be suppressed and we gave a numerical evidence of dynamical efficiency of the absorber for different types of load distributions. In the prospective work it will be of interest to develop a similar procedure for damped continuous absorbers, i.e. absorbers with viscoelastic layer, and to discuss the influence of nonlinearity.

References

[1] J.B. Hunt, Dynamic Vibration Absorbers, Mechanical Engineering Publications Ltd., London, 1979.
[2] J.P. Den Hartog, Vibracije u mašinstvu, Građevinska knjiga, Beograd, 1972.
[3] J. Ormondroyd and J.P. Den Hartog, The theory of dynamic vibration absorber, Transactions of ASME, 50, (1928), A9-A22.
[4] W.-J. Hsueh, Vibration transmissibility of a unidirectional multi-degree-of-freedom system with multiple dynamic absorbers, Journal of Sound and Vibration, 229 (4), (2000), 793-805.
[5] R.A. Burdisso and J.D. Heilmann, A new dual-reaction mass dynamic vibration absorber actuator for active vibration control, Journal of Sound and Vibration, 214 (5), (1998), 817-831.
[6] Y.M. Shi, Z.F. Li, H.X. Hua, Z.F. Fu and T.X. Liu, The modelling and vibration control of beams with active constrained layer damping, 245 (5), (2001), 785-800.
[7] G.G.G. Lueschen and L.A. Bergman, Green's function synthesis for sandwiched distributed parameter systems, Journal of Sound and Vibration, 191 (5), (1996), 613-627.
[8] T. Sakiyama, H. Matsuda and C. Morita, Free vibration analysis of sandwich beam with elastic and viscoelastic core by applying the discrete Green function, Journal of Sound and Vibration, 191 (2), (1996), 189-206.
[9] Z. Oniszczuk, Transverse vibrations of elastically connected double-string complex system, Part I: free vibrations, Journal of Sound and Vibration, 232 (2), (2000), 355-366.
[10] Z. Oniszczuk, Transverse vibrations of elastically connected double-string complex system, Part I: forced vibrations, Journal of Sound and Vibration, 232 (2), (2000), 367-386.
[11] Z. Oniszczuk, Transverse vibrations of elastically connected simply supported double-beam complex system, Journal of Sound and Vibration, 232 (2), (2000), 377-403.

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