

Contents

I APPLIED CALCULUS OF VARIATIONS AND ELEMENTS OF ANALYTICAL MECHANICS	7
1 FOUNDATIONS OF CALCULUS OF VARIATIONS	9
1.1 Introduction	9
1.2 The basic problem of the calculus of variations	12
1.3 The classical (isochronous) variation of a function	18
1.4 Necessary and sufficient conditions for an extreme value .	21
1.4.1 The Euler-Lagrange equation	23
1.4.2 The second variation	25
1.5 Some special cases of the Euler-Lagrange equation	27
1.6 Some generalizations of the basic problem	35
1.6.1 Functionals involving higher derivatives	35
1.6.2 Functionals involving several independent functions	36
1.6.3 Functionals involving several independent variables	39
1.7 More general boundary conditions	42
1.7.1 Natural boundary conditions	43
1.7.2 The problem of Bolza	46
1.8 Generalized variation	68
1.8.1 Nonsimultaneous variation	68
1.8.2 Problems with free upper boundary	72
1.9 Piecewise-smooth extremals - Erdman-Weierstrass cor- ner condition	76

2 VARIATIONAL PROBLEMS WITH CONSTRAINS	89
2.1 Constrained extrema problems - Lagrange multipliers	89
2.2 Extremizing functionals subject to algebraic constrains	93
2.3 Variational problems with differential equation constrains	98
2.4 Isoperimetric problems	116
3 CALCULUS OF VARIATIONS BY MEANS OF CANONICAL VARIABLES AND APPLICATIONS TO MECHANICS	129
3.1 Canonical variables and canonical differential equations of the calculus of variations	129
3.2 The Legendre transformation	137
3.2.1 Dual transformations	138
3.2.2 The Legendre transformation applied to the La- grange and Hamilton function	142
3.2.3 Transformations of the Euler-Lagrange equations	144
3.3 The theory of canonical transformations	145
3.4 Poisson brackets	151
3.4.1 The condition for a canonical transformation	153
3.4.2 The Poisson theorem	157
3.5 The motion of a dynamical system as a successive canon- ical transformation	159
3.6 The Hamilton-Jacobi method	165
3.6.1 The method of integration for time independent Hamiltonian	169
3.6.2 Systems with cyclic coordinates	170
3.7 Relationship between the Hamilton-Jacobi theory and canonical transformations	182
3.8 Lagrange's variation of constants method	184
4 DIRECT METHODS OF CALCULUS OF VARIATIONS	189
4.1 Introduction	189
4.2 Galerkin's method	190
4.3 Applications of Galerkin's method to partial differential equations	197

4.4	Applications of Galerkin's method to nonstationary heat conduction in solids	204
4.5	On direct methods based on Hamilton's principle	212
4.6	Variational principles in the heat conduction theory	228
4.6.1	Variational principle of Rosen - method of local potential	229
4.6.2	Variational principle of Bateman	230
4.6.3	Variational principle with a vanishing parameter .	231
5	CONSERVATION LAWS OF DYNAMICAL SYSTEMS	239
5.1	Introduction	239
5.2	Transformational properties of Lagrange-D'Alembert's principle	241
5.2.1	The energy integral	245
5.2.2	The cyclic integral	246
5.2.3	The quadratic conservative law of a nonconservative system	247
5.3	Conservative laws of reo-linear oscillators	252
5.3.1	The Lewis invariant and the adiabatic invariants .	258
5.3.2	Case of a damped linear oscillator	260
II	FUNDAMENTALS OF OPTIMAL CONTROL THEORY	271
6	THE VARIATIONAL APPROACH TO OPTIMAL CONTROL PROBLEMS	273
6.1	The basic terminology of the optimal control theory . . .	273
6.2	The mathematical description of the problem	274
6.3	Some optimization methods	278
6.4	Optimization problems for dynamic systems	279
6.5	The optimization without constrains on the control variables	282
6.6	The canonical form of the problem and Pontryagin's maximum principle	287

6.7	Continuous systems: some state variables specified at an unspecified terminal time	298
6.8	The Bolza problem and its solution	308
6.9	The Bolza problem with state variable equality constraints at boundary	311
6.10	The Bolza problem with unspecified terminal times	317
6.11	Follow and connect problems	323
6.12	The Mayer problem	331
6.13	The reduction of the Lagrange problem to the Mayer problem	334
6.14	Linear systems with quadratic criteria linear feedback	355
6.15	Continious optimal control problems with inequality constraints on state variables	363
6.15.1	The brachistochrone problem with a state variable inequality constraint	366
6.15.2	Some particular cases	375
7	OPTIMAL CONTROL WITH CONTROL VARIABLE INEQUALITY CONSTRAINTS	379
7.1	Pontryagin's maximum principle and control inequality constraints	379
7.2	Some fundamental problems	386
7.2.1	Linear optimization problems: "bang-bang" control	387
7.2.2	Minimum fuel systems	388
7.2.3	Minimum time systems	389
7.3	Extremal paths in the phase space	405
7.4	Singular solutions of optimization problems	433
8	FUNDAMENTALS OF DYNAMIC PROGRAMMING THEORY	457
8.1	Introduction	457
8.2	Optimal trajectories for multistage discrete processes	460
8.3	Applications of the principle of optimality to continuous problems and the Hamilton-Jacobi-Bellman equation	464
8.4	Remark on boundary conditions	470
	Bibliography	477

Index

- A Adiabatic invariant259 of a harmonic oscillator259 of Mendelshtam's oscillator266 of a simple pendulum263 of a variable length pendulum265 Aerodynamical problem of Newton13, 84 for a slender body35 Action integral9 Aris457 Atanackovic453
- B Bang-bang control387, 423 Bate-man230 Bellman457 Bernoulli12 Bessel's oscillator262 Billiards problem83 Biot238 Bolza47 Bolza problem in optimal control theory281, 308, 311, 317 in variational calculus47 Boundary value problem27, 39, 112, 189 Brachistochrone problem classical12, 32, 375, 455 for multistage discrete processes460 in a fluid flow307 in optimal control theory302 on a cylinder60 with a state variable inequality constraint366 with friction108, 113, 452 with initial velocity114 Bubnov196
- C Canonical variables130 Canonical transformation145 condition153 identical162 inverse162 Canonical action integral133 Canonical form of optimality conditions290 Carslaw236 Cascade integrator314 Catenary31, 123 minimal surface of revolution31 Cauchy problem for ODE39 Chaplygin problem106 Chemical reactor15, 65, 336, 337, 342 Chernousko401 Conservation law 28, 157, 247, 250 approximative262 for reo-linear oscillators253 the gas discharge problem252 Criterion function280 Cyclic integral28, 246 Cycloid33, 455
- D D'Alembert240 Direct methods of calculus of variations190 Djukic237, 453 Dual function139 Dual transformations138 Duffing186 Duffing equation186, 204, 222
- E Eigenvalue problem121, 197, 218 Elas-tic beam equilibrium configuration48 longitudinal oscillations348 longitudi-nal nonlinear oscillations53, 203, 227 transversal oscillations55, 200, 201, 350 Electrochemical process296 Emden's equation249 Erdman76 Erdman-Weierstrass corner conditions76 Euler12 Euler-Lagrange equation for several independent vari-ables41 for several independent func-tions37 of arbitrary order36 of second orderf24, 213 Evans448 Extremal path18, 24 in phase space277, 405 with corners76
- F Fermat's principle34 Filin202 Fletcher400 Follow and connect problem323, 326 Functional13, 280
- G Galerkin190 Galerkin's method190, 197 Generalized Hamilton's equation183 Generalized (nonsimultaneous) varia-tions68 Generators of canonical trans-formations147 Generators of infinit-en-simal transformations244 Geodesic line 12 Goddard446 Goddard's rocket prob-lem446 Gradient function243
- H Hamilton10 Hamilton's function (Hamil-tonian)131, 289, 469 Hamilton's prin-ciple10, 38, 212, 216, 228, 234 Hamilton-Jacobi equation167 method165, 253 Hamilton-Jacobi-Bellman equation464, 468 Heat conduction, 228 nonlinear194, 237 non-

stationary52, 204 simple one-dimensional45
I Impulsive control438, 450 Integral of energy29, 133, 245 Isoperimetric problem 116 classical120
J Jacobi153 Jacobi identity153 Jacobi theorem168 Jacobson400 Jaeger236
K Kauderer227 Kepler problem174 Ko-briski A. A268 Kobriski A. E.268 Krilov355, 401
L Lagrange12 Lagrange-D'Alembert principle240, 241 Lagrange's gyroscope135, 181 Lagrange's multipliers89 Lagrange's problem of optimal control theory281 Lagrange's function (Lagrangian)38, 132 Lagrange brackets154, 159 fundamental155 Leibniz12 Legendre26 Legendre condition26 Legendre transformation137 Lewis259 Lewis invariant259 Linear regulator problems355, 475 Lunar soft-lending problem421
M Mayer281 Mayer problem in optimal control theory281, 331, 346, 352 Mayne400 McDanell450 Meditch423 Mendelstam266 Mendelstam's oscillator266 Method of Bubnov-Galerkin196 of Kantorovich226 of local potential230 of partial integration197 of Ritz212, 221 of successive approximations398 Moving particle in a central force field174 in gravity field 172 on a cylinder95 on a inclined plane268
N Natural boundary conditions44, 65, 75 Newton12 Newton's curve87 Noether240 Noether's theorem240 Noether's identity240, 244
O Optimal control closed loop problems277 maximum distance ski flight paths351 minimum fuel problems388 minimum time problems389, 405 minimum time jet flight paths463 of a ship movement430 of a space vehicle movement326, 421, 446 open loop problems277 problem formulation276, 279 Optimal shape of a aircraft wing119
of a compressed rod103, 345, 347 of a radiator125 of a rotating rod402 of a vibrating beam348, 350 Optimal feed-back controller359 Oscillations of a chain in the vertical plane57, 225 of a wire62 with two degrees of freedom164 Oscillator damped180, 220, 247 damped - reolinear260 forced180, 181, 216, 426 linear156, 161, 163, 171, 180, 220 nonlinear186, 221, 328, 398 reolinear253
P Penetration depth206 Poisson151 Poisson equation198 Poisson's theorem157 Poisson brackets151, 159 fundamental152 Polak400 Pontryagin288 Pontryagin's maximum principle288, 291, 379 Powers450 Principle of optimality457
R Rayleigh212 Rayleigh-Ritz method212 Reeves400 Remizov355 Ritz212 Riccati179 Riccati equation179, 358 Rosen229
S Second variation22, 25 Shooting method113, 307 Simple pendulum134, 224 of variable length266 on an inclined plane265 Singular control433 Singular intervals433, 439 Snelius problem34 Spasic345 Speyer450 Sturm-Liouville equation27 Sufficient conditions for extrema25, 292 Switching function408
T Torricelli principle123 Tsien448
V Variation classical (simultaneous)18 generalized (nonsimultaneous)68 Variation of constants184 Variational principle of Bateman230 of Rosen229 with vanishing parameter232, 237 Vujanovic232, 237
W Weierstrass76
Z Zermelo problem305