International Journal of Solids and Structures 46 (2009) 2939–2949

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

# Does generalized elastica lead to bimodal optimal solutions?

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ARTICLE INFO

*Article history:*
Received 18 December 2008
Received in revised form 28 February 2009
Available online 1 April 2009

*Keywords:*
Generalized elastica
Optimization
Clamped–clamped rod

ABSTRACT

This paper answers the following question. A compressed rod clamped at both ends is assumed to rotate with a constant angular velocity. In the sense of classical Bernoulli–Euler elastica theory, the shape of the lightest rod, stable against buckling, is bimodal (i.e. associated with two buckling modes). What will be the case if we introduce more physical information in the rod model by assuming that it can suffer not only flexure but also compression and shear?



## 1. Introduction

This paper is devoted to the generalization of the Lagrange problem: to find a curve which by its revolution about an axis in its plane determines the rod of greatest efficiency. Despite its long history the problem has received much interest in recent decades. In an attempt to solve one, for different load and boundary conditions, numerous authors have turned their attention to the classical Bernoulli–Euler elastica theory and bimodal optimization. These are considered because of simplicity and because the obtained unimodal solution was not optimal having the buckling load less than the declared one, see the papers of Tadjbakhsh and Keller (1962), Olhoff and Rasmussen (1977) and Seyranian (2000).

In order to resolve the anomaly, Nikolai (1907) was the first author who proposed minimal cross-sectional area determined so that given limiting stress will not be exceeded. Much later, on the basis of an assumption that for specific boundary conditions some shapes may yield eigenvalues of multiplicity two, leading to bimodal optimization, another formulation was developed to the necessary mathematical and physical consistency, for example see Olhoff and Rasmussen (1977), Olhoff and Taylor (1983), Seyranian (1984), Barnes (1988) and Cox and Overton (1992), and the references therein. The increase in the number of papers on various aspects of the Lagrange problem, and particularly bimodal optimal solutions, is still present, see Tada and Wang (1995), Seyranian et al. (1994), Atanackovic (2001), Glavardanov and Atanackovic (2001), Seyranian and Privalova (2003), Egorov (2004), Spasic and Atanackovic (2004), Atanackovic (2006), Smaś (2007) and Olhoff and Seyranian (2008), where we have tried to keep the reference list brief. Among all possible connections, our aim will be to enquire how unimodal solution can be regularized when shear and axial deformations are imposed on the rod model.

To motivate the answer to the posed question, note that by involving the limiting stress in the Lagrange problem, as Nikolai did, it was implicitly allowed that the rod could change its length under compression. It was a rather artificial condition since the classical Bernoulli–Euler theory of buckling, neglects the axial strain of central line as a possible deformation of an elastic rod. Besides extensional rigidity, in the problem of Lagrange, the infinite value of the shear rigidity was also assumed. The first work that generalizes the classical elastica theory as to take the shearing forces into account goes back to Engesser who considered the influence of shear on the buckling loads in 1889. Pflüger analyzed the influence of the axial strain on the stability of a simply supported compressed rod, see Pflüger (1975). It is well known fact that the Euler buckling load is sensitive to both effects: decreasing of shear rigidity of the rod the value of the Euler buckling load decreases, and decreasing of extensional rigidity the value of critical load increases. In the seventies of the former century many generalized elastica theories taking into account both shear and axial strain were proposed. Just a few examples from the voluminous literature are Schmidt and DaDeppo (1971), Reissner (1972), Goto et al. (1990) and Atanackovic and Spasic (1994), for plane and Kingsbury (1985), Eliseyev (1988) and Simo and Vu-Quock (1991), for spatial problems. The aim of any theory of rods is to characterize the deformed configuration of a slender three-dimensional body by a single curve and certain parameters recording material orientation to that curve, Parker (1979). Each of them must necessarily be approximate, having a chance to be more appropriate in specific applications then the others, Atanackovic (1997). The influence of finite values of extensional and shear rigidities on the optimal shape of elastic rods, was treated in Spasic (2002). By use of the constitutive equations that take into account

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doi:10.1016/j.ijsolstr.2009.03.019