

SERBIAN ACADEMY OF ARTS AND SCIENCES, BRANCH IN NOVI SAD

**INTERNATIONAL SYMPOSIUM ON NONCONSERVATIVE AND
DISSIPATIVE PROBLEMS IN MECHANICS**

DEDICATED TO 75 YEARS OF PROFESSOR BOŽIDAR VUJANOVIĆ

11-14 September 2005, Novi Sad, Serbia

Serbian Academy of Arts and Sciences, branch in Novi Sad, (Platoneum)

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INTERNATIONAL SYMPOSIUM ON NONCONSERVATIVE AND
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Academician

BOŽIDAR D. VUJANOVIĆ

Professor Božidar D. Vujanović was born in Smederevo (Serbia) in 1930. He graduated at the Department of Mechanics at the University of Belgrade 1955 and his doctorate was conferred to him at the same University at 1963 from the area of mechanics. He was employed at the Mechanical Engineering Faculty at the University of Belgrade as an Assistant of Mechanics from 1957 to 1963. Since then he has been at the University of Novi Sad - Department of Mechanics at the Faculty of Technical Sciences and spent there all the time until he retired at 1995. Between 1967 and 1969 he was in the USA as a Re-



search Associate at the University of Kentucky. From 1977 to 1978 he visited Japan as Visiting Professor at the University of Tsukuba and at 1984 he spent six months as a Visiting Professor at the Department of Mechanical and Material Engineering at the Vanderbilt University in Nashville, USA.

The scientific interest of Professor Vujanović is Theoretical and Applied Mechanics, Variational Principles and their applications to conservative and non-conservative dynamical systems, heat conduction theory, optimal control theory, nonlinear oscillations with dissipative elements etc.

In 1990 Professor Vujanović became a Correspondent Member of the Academy of Sciences and Arts of Vojvodina in Novi Sad, and after the fusion of this Institution with the Serbian Academy of Sciences and Arts in Belgrade he has been adopted as the member of the same rank. At 2000 he has been elected as the Full Member of the Serbian Academy of Sciences and Arts in Belgrade.

⁰NOVI SAD, SERBIA, SEPTEMBER 11-14, 2005

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I. Müller

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Phase diagrams modified by interfacial penalties

The conventional forms of phase diagrams are constructed without consideration of interfacial energies and they represent an important tool for chemical engineers and metallurgists. If interfacial energies are taken into consideration, it is intuitively obvious that the regions of phase equilibria must become smaller, because there is a penalty on the formation of interfaces. We investigate this phenomenon qualitatively for a one-dimensional model, in which the phases occur as layers rather than droplets or bubbles. The modified phase diagrams are shown.

⁰NOVI SAD, SERBIA, SEPTEMBER 11-14, 2005

T. Ruggeri

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Acceleration waves and weak Kawashima-Shizuta condition

We consider dissipative hyperbolic systems of balance laws in which a block of equations are conservation laws which arise typically in the Extended Thermodynamics [1]. In this case, a coupling condition firstly introduced by Kawashima-Shizuta (K-condition) [2], play a fundamental role for the global existence of smooth solution for small initial data and for the stability of constant state [3]-[5]. Nevertheless the global existence example by Zeng [6] prove that the K-condition is only a sufficient condition.

From physical point of view it is important to understand which is the minimal part of the K-condition that is also a necessary condition in such way we can use this as selection rule for the admissible production terms.

Using acceleration waves we can verify that is a necessary condition a weaker condition in which the K-condition is required only for the genuine non-linear characteristic velocities and not for the linear degenerate one [7].

This weak K-condition is unfortunately not sufficient for generic global smooth solutions and the problem to determine new conditions to add to the weak K-condition is still an open problem.

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G.C. Nolen and A.M. Strauss

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**The direct employment of fission fragment energy for a hybrid
space thruster**

Space propulsion produced by the energy from nuclear fission fragments provides immense opportunities for efficient deep space and interplanetary exploration. In order to take humankind further into space than current technologies permit, a thruster must be developed that is capable of high thrust as well as have the ability to operate for long periods of time. Such designs will reduce trip time and provide the versatility necessary for visiting other planets, moons, etc. A fission fragment thruster system that has this potential for flexible deep space missions is the hybrid fission fragment thruster (HFFT). The HFFT is composed of two different but complementary systems. The HFFT will operate with a low thrust (1.35 N) directed fission fragment thruster (DFFT) and a high thrust (745,000 N) Chemical Fission Fragment Thruster (CFFT). In the directed fission fragment design, Americium 242m fissions in a vacuum and the positively charged fission products are immediately used for propulsion by directing them out of the reactor on magnetic field lines created by exterior solenoids. The second system operates by converting liquid hydrogen, methane, or ammonia to a high temperature gas as it flows through tubes lined with the same fissionable material as the DFFT. The fission fragments pass through the liquid, directly transferring their kinetic energy to the hydrogen. The expanding gaseous hydrogen then passes through a conventional rocket nozzle in order to provide high thrust. The performance of this coupled system and possible mission applications provide the scope of this paper.

⁰NOVI SAD, SERBIA, SEPTEMBER 11-14, 2005

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Extragalactic celestial mechanics: an introduction

With reference to the present era for the Universe (the matter-dominated era), the Cosmological Principle (the Universe is homogeneous and isotropic) and the principles of classical mechanics imply the following main results, without resorting to any theory of gravitation:

1. there exist three possible geometries for the Universe \mathfrak{U} : the geometry of a three-dimensional sphere S^3 , the geometry of a three-dimensional pseudosphere PS^3 and the geometry of the ordinary Euclidean space E^3 ;
2. Hubble's law;
3. the equation of evolution of \mathfrak{U} which is the same in all three cases S^3 , PS^3 , E^3 ;
4. in case E^3 , Newton's law of gravitation which holds true for all \mathfrak{U} ;
5. the existence of two constants for \mathfrak{U} : one is the gravitational constant G and the other is the cosmological constant Λ ;
6. if $\Lambda = 0$, the study of the motion of any typical galaxy with respect to another one can be reduced to the two-body problem of the classical celestial mechanics;
7. the explicit expression of the Hubble parameter in terms of the density $\mu(t)$ of \mathfrak{U} and of a constant a (never considered before) which may be determined by astronomical observation;
8. the equation that permits us to establish the age of the Universe;
9. the energy constant α which is equal to 0 only in case E^3 and viceversa;
10. in case E^3 two completely determined models for \mathfrak{U} follow: one with $\Lambda \neq 0$ and the other with $\Lambda = 0$; the second one is the celebrated Einstein-de Sitter model of \mathfrak{U} , always obtained until now by working within the framework of the general theory of relativity;
11. when $\alpha \neq 0$ the constant α cannot be determined starting from the assumptions made and the astronomical observation: this represents the extreme boundary which can be reached starting from the two assumptions made.

For going farther it is necessary to make recourse to another assumption, which is the well known property of the velocity of light of being independent of its source. With this new assumption it follows:

12. in the case of the Universe this new assumption does not contrast with the second assumption made;
13. all the results obtained in cosmology for the propagation of light in the Universe follow;
14. the determination of the constant α : $\alpha < 0$ characterizes the case S^3 , and $\alpha > 0$ the case PS^3 ;
15. from the three assumptions made all the pressureless homogeneous isotropic expanding models of the Universe follow;
16. for all the equations obtained there exists one and only one tensor form which is given by Einstein's field equations of general theory of relativity;
17. all the above deductions have nothing to do with the so-called "Newtonian cosmology" which, even if it appears in many books on general relativity and cosmology, is untenable.

V. Djordjević

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**On a model equation that reflects some of the shear flow
hydrodynamic stability properties**

Very important problems of laminar-turbulent transition processes in fluid mechanics are described by the hydrodynamic stability theory. This theory is mathematically delicate and perplex, and algebraically tedious and involved, so that very often important physics of the problem is hidden behind the complex mathematical operations. That is why there exist in the literature several so-called model equations that are physically not directly related to any of the problems of fluid mechanics, but mimic very accurately hydrodynamic stability properties of various flows, which can be revealed by much simpler mathematical methods.

One of such equations is proposed in this paper. It contains the basic velocity profile, whose stability properties are investigated, and is particularly suited for studying these properties for free and bounded shear flows. Both linear and weakly nonlinear theories of this equation are developed and presented. It is shown within the linear theory that the unbounded jet type velocity profile experiences long wave instability, and that the eigenfunctions are expressed in terms of associated Legendre functions. Within the weakly nonlinear theory neutral eigenmode is perturbed by introduction of some slowly varying independent variables, and a Landau type equation is derived, which describes the long time evolution of this mode. The conditions for the appearance of supercritical stability and subcritical instability are illuminated.

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**A convenient way to solve 2PBVP in canonical form:
Construction of the Hamilton principal function for
rheo-linear systems**

If the principal function W is given, the entire dynamical problem is reduced to differentiations and eliminations.

CORNELIUS LANCZOS

The Hamilton-Jacobi method is briefly summarized and a complete integral to the Hamilton-Jacobi equation, based on the particular solution of the Riccati equation, is used to construct the Hamilton principal function for arbitrary rheo-linear systems with a single degree of freedom. It appears that explicit principal function is a quadratic form of the generalized coordinate at initial and terminal state. The principal function is then used to solve the canonical differential equations for given two-point boundary conditions. The method is successfully applied to a number of examples from a class of nonconservative dynamical systems.

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Nonlinear distributed order fractional differential equations

We study the existence and uniqueness of solutions of equations of the form

$$ay^{(2)}(t) + \int_0^2 \phi(\alpha) y^{(\alpha)}(t) d\alpha = f(y, t).$$

Since for $y \in L^1_{loc}([0, \infty))$,

$$\alpha \rightarrow y^{(\alpha)} \quad \text{is a smooth mapping } \mathbb{R} \rightarrow S',$$

one can take $\phi(\alpha) = \delta(\alpha)$ and obtain (with $a = 0$), as a special case, equation $y^{(\alpha)}(t) = f(y, t)$, $\alpha \in (0, 1)$, which was studied by several authors.

Such equations arise in the distributed derivatives models of viscoelasticity and system identification theory.

It is a part of joint work with coauthors.

R. Gorenflo

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**On the distinguished role of the Mittag-Leffler function
in fractional relaxation and in renewal processes with
power law waiting time**

The entire transcendental function $E_\beta(z) = \sum_{k=0}^{\infty} z^k / \Gamma(1 + k\beta)$, introduced in the year 1903 by G. Mittag-Leffler and named after him (see [2] for a very good survey of its properties) and some of its generalizations in recent two decades gradually have come to be recognized as useful in diverse applications. Some such applications have been discovered only in the Laplace transform domain, see [1] and [3], the authors being unaware of the fact that in the time domain they reduce to the closely related functions that we take into view in this lecture, namely, for $0 < \beta \leq 1$, $t > 0$, the variants $\Psi_\beta(t) = E_\beta(-t^\beta)$ and $\psi_\beta(t) = -d[\Psi_\beta(t)]/dt$.

Both functions are completely monotone and appear in the standard process of fractional relaxation (see e.g. [4]), the modelling being done by the Caputo or by the Riemann-Liouville fractional derivative. They also play decisive roles as waiting time probability distribution or density in the theory of continuous time random walks (see e.g. [5], [6], [8]) and appear as the limiting probability law in infinite thinning of a renewal process for an initial density with power law asymptotics. We outline these models and show furthermore that the Mittag-Leffler waiting time law is the precise description of the properly rescaled long time behaviour of a renewal process whose waiting time density has a long tail due to a power law decay at infinity. In this sense the Mittag-Leffler process, generalizing the classical Poisson process and described in [7] and [8], is an important limiting process. In the analysis of such asymptotic behaviour the Mittag-Leffler density $\psi_\beta(t)$ exhibits stability against a rescaling procedure combined with a deceleration and a characteristic kind of self-similarity. Remarkably there arises the same transformation formula in the analysis of infinite thinning and of long time behaviour of a power law renewal process. The results presented in this lecture have been obtained in collaboration of the speaker mainly with F. Mainardi.

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**Mathematical and physical aspects of fractional
calculus in linear viscoelasticity**

A topic of continuum mechanics, where fractional calculus is suited to be applied, is the linear theory of viscoelasticity. In fact, in the first half of the past century, a number of authors have, *implicitly* or *explicitly*, used fractional calculus as an empirical method of describing the properties of linear viscoelastic materials. In 1971, extending earlier work by Caputo, Caputo and Mainardi [4] suggested that derivatives of fractional order could be successfully used to model the dissipation in seismology and in metallurgy. Since then up to nowadays, applications of fractional calculus in different areas of rheology have been considered by several authors; in this respect let us point out the reviews by the Author [8, 9], by Rossikhin & Shitikova [10], and the recent lecture notes by Atanackovic [2]. The purpose of this lecture is twofold: from one side we attempt to provide a physical interpretation of the viscoelastic models constructed via fractional calculus and from the other side we intend to analyze some mathematical approaches to fractional calculus adopted up to nowadays in linear viscoelasticity. We start from the classical linear hereditary theory of continuum mechanics, where in many cases the coupling of thermo-elastic effects is responsible of memory phenomena that are mathematically described by integro-differential constitutive equations between stress and strain. By generalizing a classical argument by Zener, the adoption of the fractional calculus allows us to describe the "fractional diffusion" responsible of long memory effects, in particular the power-law decay of the relaxation function in the generalized Standard Linear Solid [7]. We consider the effects of the initial conditions in properly choosing the mathematical definition for the fractional derivatives that are expected to replace the ordinary derivatives in the constitutive equations of spring-dashpot models. Finally, we revisit the approaches by Kempfle [5, 6] and Atanackovic [1, 3].

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**New trends in active systems interacting
with dynamic environments**

In principle, one can distinguish two basic subclasses of contact tasks. One subclass essentially covers force tasks, whose very nature requires an end-effector to establish physical contact with the environment and exert a process-specific force. In general, these tasks require both the position of the end-effector and the interaction force to be controlled simultaneously. A typical example of such tasks is machining processes such as grinding, deburring, polishing, bending, etc. In these tasks, force is an inherent part of the process and plays a decisive role in its execution (e.g., metal cutting or plastic deformation). In order to prevent overloading or damaging of the tool during the operation, force must be controlled in accordance with some definite task requirements.

The prime emphasis of the tasks from the other subclass lies on the end-effector motion that has to be realized close to the constrained surfaces (compliant motion). A typical representative of such tasks is the part mating process. The problem of controlling the robot during these tasks is, in principle, the problem of accurate positioning. However, due to imperfections inherent in the process, sensing, and control system, these tasks are inevitably accompanied by the occurrence of contact with the constrained surfaces, which results in the appearance of reaction forces. The measurement of interaction force provides useful information for error detection and appropriate modification of the prescribed robot motion.

The future will certainly hold more tasks for which interaction with the environment is of fundamental importance. Recent medical robot applications in surgery, e.g., spine surgery, neurosurgical and microsurgical operations, and hip implant operations, could also be considered essentially as contact tasks. Comprehensive research programs in automated construction, agriculture, and the food industry, focus on the robotization of several representative contact tasks, such as underground excavation, meat deboning, etc.

In the following text we will present in brief several examples of contact tasks that are currently occupying the attention of researchers and professionals, and whose common specific feature is the contact of the system (structure) with its dynamic environment.

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**On the lateral deformation of an elastic rod with step
change in cross-section and arbitrary transversal
loading**

The lateral deformation of an elastic rod is studied within the framework of distribution theory. It is assumed that the rod can have step discontinuity in the cross-section and that it is loaded by concentrated axial forces and distributed transversal forces. The existence and properties of the solution is studied. The main characteristic of our approach is the use of a *single* second order differential equation valid in whole interval of interest (including points of discontinuities). From this equation, that we call governing equation, the solution in any point of the rod interval, including jumps at the points of discontinuities, can be obtained.

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Optimum design of structures subjected to follower forces

The shape optimization is used to maximize the flutter load of an *Euler-Bernoulli* cantilever column having fixed mass. The column may be submitted either to a constant follower force P at its tip or to an arbitrarily distributed follower force $p(x)$ along the length of the column. The optimum design of Beck's and Hauger's columns has been investigated by several researchers using semi-analytical [1] or FEM-based numerical [2] methods. The obtained, however, results are restricted to columns with similar cross sections, namely constrained by the relation $I(x) = aA(x)^n$, ($n = 1, 2, 3$), with $A(x)$ being the area of the cross-section, $I(x)$ its moment of inertia and a a specified constant. Moreover, all these solutions ignore any restrictions that should be imposed on the rate of change of the cross section to ensure the validity of the *Euler-Bernoulli* theory, as well as lower bounds resulting from serviceability reasons. Thus, the obtained optimum solutions are often unrealistic.

In this investigation a new solution procedure to the optimization of the flutter load is developed. This is achieved by distributing appropriately the stiffness and mass of the beam along its length so that the critical load takes the maximum or a prescribed value. The critical load is obtained from the relation $P_{cr} = \tilde{n}_0^l s \tilde{v} \tilde{u} dx / \tilde{n}_0^l v \tilde{u} dx$, where u, v are the solution of the direct and the adjoint eigenvalue problems, respectively, and is the bending stiffness. While no restriction is imposed on the variation law of the beam stiffness and mass properties, the proposed method respects the above mentioned restrictions concerning the validity of the *Euler-Bernoulli* beam theory and the serviceability of the column. The problem is reduced to a nonlinear optimization problem under equality and inequality constraints as well as specified lower and upper bounds. The proposed method is workable thanks to the capability to evaluate the objective function, which requires the solution of the eigenvalue problems (direct and adjoint) for the beam equation with variable stiffness and mass subjected to an arbitrarily distributed axial follower force. This problem is solved using the analog equation method (AEM) as developed for the fourth order differential equation with variable coefficients [3,4]. Besides its accuracy, this method overcomes the shortcoming of a possible FEM solution, which would require resizing of the elements and re-computation of their stiffness properties during the optimization process.

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Stability of motion of a cam-follower system

In this paper the stability of motion of a cam-follower system which is the vital part of a cam mechanism is analyzed. The cam mechanism represents one of the most widely applied motion transmitters. The rotating motion of the cam is transformed into straight forward motion of the follower. Due to elasticity of the system and vibrations of the camshaft and the follower the transmission ratio between cam and follower is perturbed. In this paper the influence of the torsional vibrations of the camshaft and axial vibrations of the follower on the accuracy of transmission ratio is investigated. The cam-follower system has two degrees of freedom and is mathematically modeled with two ordinary coupled second order parametrically excited non-linear differential equations. The parametric resonance of difference type in the presence of one-to-two internal resonance is treated. The method of multiple scales is used to derive four first-order autonomous ordinary differential equations for the modulation of the amplitudes and phases. The steady state solutions of the modulated equations and their stability are investigated. The trivial solution loses the stability through bifurcation giving rise to non-trivial solution. Using the analytical results the amplitude-modulated motion for a certain cam curve profile is numerically obtained. The numerical results are in good qualitative agreement with theoretical prediction.

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**On Noether's theorem for non-linear anisotropic
elastic materials**

The problem how to represent anisotropic tensor functions, i.e. how to deal with constitutive equations of anisotropic materials, was pushed a head, for instance, by the papers of Boehler (1978, 1979). In these papers an anisotropic tensor function is expressible as an isotropic one with structural tensors as the additional tensor agencies (see also Liu, 1982). As pointed out by Q.S. Zheng (1994), there are three potential additional significant benefits of this idea in formulating constitutive equations of anisotropic materials.

First, this concept allows constitutive equations to be formally expressed in isotropic forms irrespective of the actual anisotropy of materials in consideration.

Second, the effects of anisotropy in constitutive equation become more clear via structural tensors.

Third, the constitutive equation is coordinate-free.

Now we add the fourth benefit: Making use of this concept, we are able to derive conservation laws for some classes (for all, Xsiao, 1995) of non-linear anisotropic materials in the same way as we do for isotropic materials. More precisely, we may approach the problem of conservation laws for anisotropic materials using Noether's theorem as we do for isotropic non-linear materials.

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**Bifurcation of the roots of the characteristic
polynomial and destabilization paradox**

Paradoxical effect of small dissipative and gyroscopic forces on the stability of a linear non-conservative system, which manifests itself through the unpredictable at first sight behavior of the critical non-conservative load, is studied. By means of the analysis of bifurcation of multiple roots of the characteristic polynomial of the non-conservative system, the analytical description of this phenomenon is obtained. As mechanical examples two systems possessing friction induced oscillations are considered: a mass sliding over a conveyor belt and a model of a disc brake describing the onset of squeal during the braking of a vehicle.

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**Shock structure in extended thermodynamics - linear
stability and bifurcation analysis**

In hyperbolic systems of conservation laws shocks are discontinuous solutions with jumps located at the singular surface - shock front. Jumps of field variables are determined by Rankine-Hugoniot equations and can be regarded as solutions which bifurcate at points where shock speed coincide with the characteristic speed of hyperbolic system.

Dissipative mechanisms, like viscosity and heat conduction, smooth out the shock and transform it into a shock structure - continuous traveling wave profile with steep gradients in the neighborhood of singular surface. This profile is determined by a heteroclinic orbit - particular solution of the ODE system whose endpoints correspond to solution of Rankine-Hugoniot equations. Like the shocks itself, shock profiles exist only when shock speed satisfy some form of admissibility condition.

In extended thermodynamics dissipation is included through additional balance laws containing source terms with relaxation. These equations describe irreversible processes far from equilibrium. Such a system retains hyperbolic character while its characteristic speeds could be different from the characteristic speeds of equilibrium subsystem of conservation laws. It will be shown, by means of linear stability analysis, that the exchange of stability of the endpoints of heteroclinic orbit coincides with the shock admissibility criterion for underlying equilibrium subsystem. Furthermore, center manifold reduction will reveal that the appearance of shock profile follows transcritical bifurcation pattern in the neighborhood of the highest characteristic speed of the subsystem. It will be demonstrated that the same bifurcation pattern appears also in the Navier-Stokes-Fourier theory, which is essentially parabolic.

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Multiparameter stability theory with mechanical applications

Stability theory is one of the most interesting and important fields of applied mathematics having numerous applications in natural sciences as well as in aerospace, naval, mechanical, civil and electrical engineering. Stability theory was always important for astronomy and celestial mechanics, and during last decades it is applied to stability study of processes in chemistry, biology, economics, and social sciences. Since every physical system contains parameters, the main goal of the present work is to study how a stable equilibrium state or steady motion becomes unstable or vice versa with a change of problem parameters. Thus, the parameter space is divided into stability and instability domains. It turns out that the boundary between these domains consists of smooth surfaces, but can have different kind of singularities. One of the motivations and challenges of the present work is to bring some qualitative results of bifurcation and catastrophe theory to the space of problem parameters making the theory also quantitative, i.e., applicable and practical. It is shown how the stability boundary and its singularities can be described using information on spectrum of the system at regular and singular points of the boundary. A new multiparameter bifurcation theory of eigenvalues of matrix and linear differential operators is presented which is a key point for stability study of systems with finite degrees of freedom as well as distributed systems. Two important cases of strong and weak interactions (collisions) are distinguished and geometrical interpretation of these interactions is given. The presence of several parameters and the absence of differentiability of multiple eigenvalues constitute the main mathematical difficulty of the analysis. We could overcome this difficulty studying bifurcations of eigenvalues along smooth curves in the parameter space emitted from singular points of the stability boundary. With presented multiparameter bifurcation theory of eigenvalues we analyze singularities of stability boundaries and give a consistent description and explanation for several interesting mechanical effects like gyroscopic stabilization, flutter and divergence instabilities, transference of instability between eigenvalue branches, destabilization and stabilization by small damping, disappearance of flutter instability, parametric resonance in periodically excited systems etc.

A significant part of the work is devoted to difficult stability problems of periodic systems dependent on multiple constant parameters. This subject has been a challenge for more than one hundred years since Mathieu, Floquet, Hill, Rayleigh, Lyapunov, Poincare. From the very beginning these problems were multiparameter. In the present work, with the bifurcation theory of multipliers, geometrical description of the stability boundary and its singularities for periodic systems is given. Then we formulate and solve parametric resonance prob-

lems for one- and multiple degrees of freedom systems in three-dimensional space of physical parameters: excitation frequency, amplitude, and viscous damping coefficient assuming that the last two parameters are small. The main result obtained here is that we find the instability (parametric and combination resonance) domains as half-cones in three-parameter space with the use of eigenfrequencies and eigenmodes of the corresponding conservative system. Finally, stability boundaries for non-conservative systems under small periodic excitation are investigated. As applications of the presented theory, we consider a number of mechanical stability problems including pipes conveying fluid, beams and columns under different loading conditions, rotating shafts and systems of connected bodies, panels and wings in airflow etc. For these systems we perform the detailed multiparameter stability analysis showing how the developed bifurcation and singularity theory works in specific problems. The experiments on parametric resonance of elastic beams done in cooperation with professor H. Yabuno and his group confirm accuracy of the obtained theoretical results.

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Stability of a pipe on nonlinear elastic foundation

The aim of this paper is to study stability of a pipe, resting on a nonlinear elastic foundation, through which a string is pulled. Stability analysis is performed by the Ljapunov-Schmidt method. Stability boundary is found from the eigenvalues of the linearized problem. Also the type of bifurcation is examined. For constitutive equation the classical Bernoulli-Euler theory is used.

⁰NOVI SAD, SERBIA, SEPTEMBER 11-14, 2005

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On the gauge-extended equations in optimal control

In this work we propose one generalization of optimal control equations, which is based on the gauge-invariant (invariant up to the gauge term) properties of suitable Lagrange and Hamilton functions with finite number of degrees of freedom. To obtain such equations additional arbitrary gauge function has been added in lately defined generalized momenta, keeping canonical structure of equations. Although this function invokes new effects in the generalized system operability in integration of such system, could be significantly increased. Consequently, the associated Hamilton-Jacobi-Bellman (HJB) equation is gauge extended as well.

Our study is focused on dynamical system, in which components of control vector are not submitted to inequality constraints, although study of these processes might be possible in certain circumstance.

⁰NOVI SAD, SERBIA, SEPTEMBER 11-14, 2005

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**Analysis of impact dynamics by use of
the regularized model approach**

In recent years there has been a significant increase in modeling systems characterized by unilateral constraints, impacts and impulsive control actions. This increase is motivated by numerous applications. In order to get responses that are very close to what experiments show several approaches working with an ideal rigid-body model before and after impact are developed. This is because the rigidity assumption causes problems connected with existence and uniqueness of solutions, as well as with energy balance laws. We have in mind the general oblique and rough impact [1], the colinear impact of three spheres [2], and the Painleve problem [3]. By use of so-called regularized models in which the contact area is substituted by a massless spring-damper element, the response of the system during impact interval is computed by solving the corresponding differential equations [4]. In order to regularize problems in both mathematical and thermodynamical point of view, instead of massless spring-damper elements one can use the constitutive equations of viscoelastic body that comprises fractional derivatives of stress and strain and the restrictions on the coefficients that follow from Clausius-Duhem inequality [5]. We shall show how the collision of a point and a fixed plane, the colinear impact of three spheres, and the Painleve problem can be regularized in such way.

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⁰NOVI SAD, SERBIA, SEPTEMBER 11-14, 2005